UNIT II

Chapter 2 - The Variables of Music

In this chapter the basic variables of music will be examined for their mathematical content. Since the general reader cannot be expected to have a background in musical notation, musical examples will be translated into graphic form. Explanations have been simplified to avoid specialized musical terminology wherever possible. When musical terms are used they are clearly defined and graphically interpreted.

"Let's start at the very beginning! A very good place to start, .•. " And with those words we launch into one of the most delightful songs to come out of the musical theater; the "Do-Re-Mi" song from The Sound of Music by Rodgers and Hammerstein.**1** (S12) (L1) The familiar lyrics of the last refrain are as follows: http://en.wikipedia.org/wiki/Do-Re-Mi

Doe a deer, a female deer,
Ray a drop of golden sun,
Me a name I call myself,
Far a long, long way to run.
Sew a needle pulling thread,
La a note to follow sew,
Tea a drink with jam and bread
That will bring us back to doeI
Do-re-mi-fa-so-la-ti-doI

http://www.culturalmath.com/media/Sound-12.mp3

Scales

As almost everyone knows this song was used by Maria to teach the Trapp family children the elements of singing in tune. The basic method, known as solfege, has a long history. Essentially, it associates a syllable with each tone of the scale. Rodgers' song is composed in C major, the scale that we examined in Chapter 1. The singing syllables are Do, Re, Mi, Fa, Sol, La, Ti and they are matched with the scale tones as follows, the octave repetition of C(Do) bringing the scale to a satisfactory melodic conclusion.

Scale tones:	C	D	E	F	G	A	B	c
Syllables:	Do	Re	Mi	Fa	Sol	La	Ti	Do

Notice that in the last line of the song this one octave span of the scale is actually sung. In the body of the lyrics, Hammerstein cleverly modified the traditional syllables to provide a memory device for the children and, in addition, amusing associations for Doe, Ray, Me, Far, Sew, La, Tea.

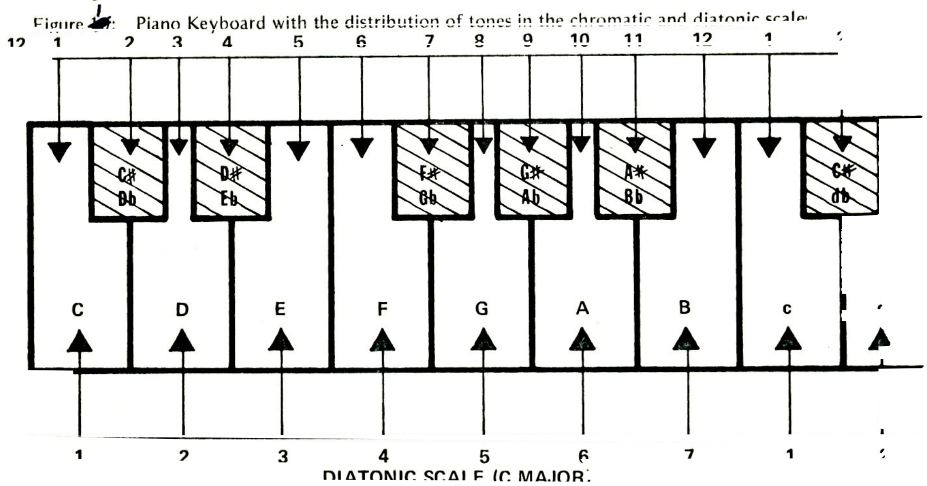
Chromatic Scale

The song also incorporates tones that are not part of the C major scale. These are sharps (#) and flats (b) that are seen as black keys on the piano keyboard (Figure 1). This extended scale is known as the chromatic scale and before proceeding to discuss the song, the structure of the chromatic scale will be explained.

The human ear is capable of hearing sounds that lie approximately between 16 to 20,000 cycles per second.

Figure

Figure 1: Piano Keyboard with the distribution of tones in the chromatic and diatonic scale



Although most of the music we listen to is rarely composed beyond 4000 cps, the sound continuum is best approximated by the sound of a siren. If this were plotted by convenient octave divisions on the horizontal axis, the frequencies would distribute themselves vertically as an exponential curve (Figure 2). For several reasons, including the nature of hearing, it is more convenient to display this relationship as a logarithmic function. This means the octave repetitions of the chromatic scale can be represented as a set of 12-inch rulers laid end to end with the units corresponding to tones in the chromatic scale (Figure 3). Each ruler provides us with a different octave presentation of the scale. The distance between units is musically called a "semitone." The piano keyboard is a very practical application of this logarithmic subdivision of the sound continuum to instrumental construction. The succession of white and black keys corresponds exactly to the units on the rulers. For readers who may still own a slide rule in this age of the electronic calculator, a comparison between the D and L scales provides an excellent analogy for the conversion of tone frequencies to their scale positions.

The piano is fixed by its system of tuning to the semitone units on the scale. Instruments like the violin can theoretically play any tone in the continuum and frequently do with good musical results, even though these tones may lie between the unit subdivisions. If, in a concert, a piano concerto is to be played, the orchestra will tune itself to the piano rather than the oboe in order to accommodate the piano's fixed tuning.

Figure

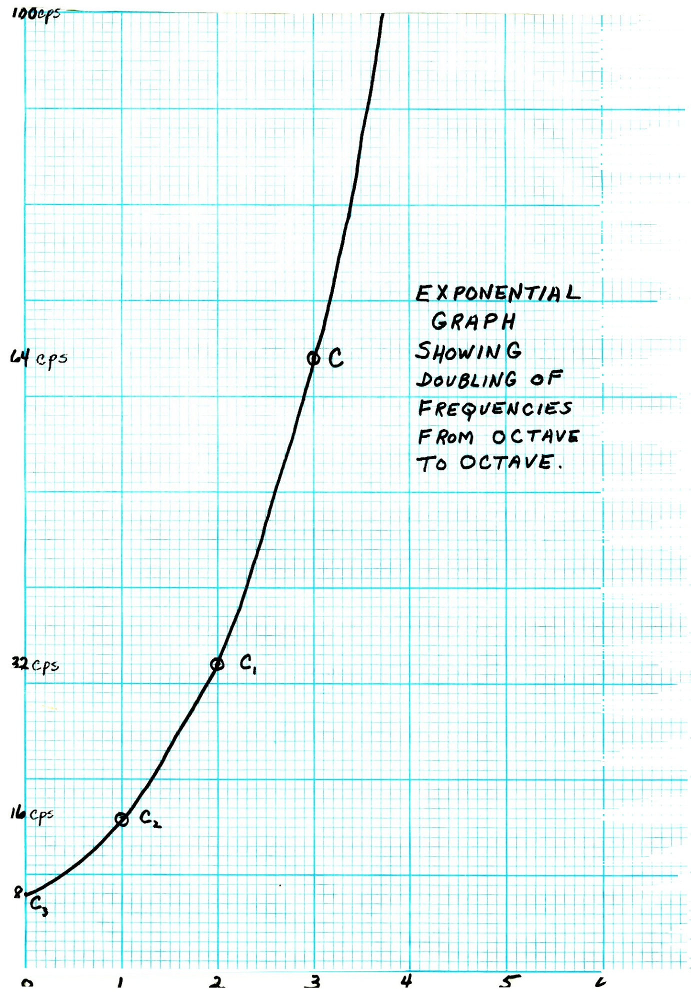
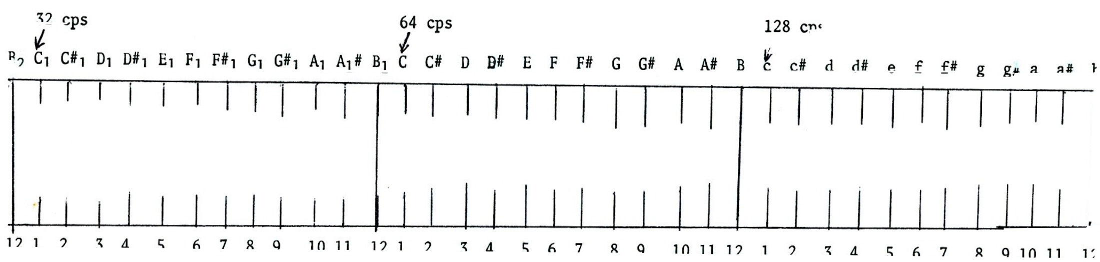

100cps
64 cps
32 cps
16 cps
8
C
C_1
C_2
C_3
EXPONENTIAL GRAPH SHOWING DOUBLING OF FREQUENCIES FROM OCTAVE TO OCTAVE.
0
1
2
3
4
5
6

Figure [illegible]

THREE OCTAVES OF THE CHROMATIC SCALE AS SEEN BY LOGARITHMS



Equal Temperament

The standard system of tuning for the chromatic scale is known as equal temperament. It was designed as a compromise solution to meet the needs of composers in modulating freely to different scales. This was a necessity that arose with the development of harmony. Mathematically the system was dependent upon the invention of logarithms by the Scottish mathematician, John Napier (1550-1617).
http://en.wikipedia.org/wiki/John_Napier

Mersenne was the first to expound the derivation of the scale in 1635. http://en.wikipedia.org/wiki/Mersenne

It primarily depends upon finding the 12th root of 2. Today this is readily approximated by a scientific calculator. The equations are as follows:

Let x = The 12th root of 2 (or 2 to the 1/12 power)

Log x = (1/12)log 2.

The log 2 = 0.3010.

(1/12) log 2 = .025086 = Log x.

Therefore, finding the anti-log, x = 1.059463. A chromatic scale can now be constructed by starting with any arbitrary tone and multiplying or dividing its frequency by powers of 1.059463. This is shown in the table below and compared to the frequencies of the Pythagorean scale derived through the sequence of fifths in Chapter 1.

Let the constant, t = 1.059463 in the table.

Chromatic Scale Tone	Letter Designation	Pythagorean Frequency	Equal Tempered Frequency	Derivation
1	C	260.8	261.6	$440/ t^9$
2	C# (Db)		277 .2	$440/ t^8$
3	D	293.3	293.7	$440/ t^7$
4	D# (Eb)		311.1	$440/ t^6$
5	E	330	329.6	$440/ t^5$
6	F	347.6	349.2	$440/ t^4$
7	F# (Gb)		370	$440/ t^3$
8	G	391.2	392	$440/ t^2$
9	G#(Ab)		415.3	$440/ t^1$
10	A	440	440	
11	A#(Bb)		466.2	$440 X t^1$
12	B	495	493.9	$440 X t^2$
13	c	521.6	523.3	$440 X t^3$

In beginning the structure of the equal tempered scale above, the tone A is set to the international standard (440 cps). This is the only equality between the Pythagorean and equal tempered scales. The remaining tones are derived as indicated by multiplying or dividing 440 cps by powers of t = 1.059463. All of the derived tones show a fractional difference with their Pythagorean counterparts. In the equal tempered scale only purity of the octave ratio (1:2) is preserved. This is seen by comparing the the frequencies of C(261.6) and c(523.3), with a slight correction for rouning off to tenths. The Pythagorean scale, on the other hand, also preserves the purity of its perfect fifths.

However, if we were to continue deriving the other tones (C#(Db) , D#(Eb), F#(Gb), G#(Ab), A#(Bb)) for the Pythagorean scale by the method employed in Chapter 1, we should end up with pitch discrepancies between the enharmonic equivalents; (i.e., C# and Db are enharmonic equivalents).

For example, the Pythagorean F# would have a frequency of 371.3 cps while the Pythagorean Gb would have 366.3 as its frequency, a difference of 5 cps that would definitely be audible. To accommodate this in the harmonies of our music today would require placing two black keys, one for F# and one for Gb, between the white keys F and G on the piano, if we continued to employ the Pythagorean tuning. This is the problem that plagued the development of keyboard instruments like the organ, clavichord, and harpsichord until equal temperament was invented. To encourage adoption of this system, the great composer, Johann Sebastian Bach (1685-1750), composed two sets of keyboard compositions. These works entitled The Well-Tempered Clavier are built in major and minor keys for every step in the chromatic scale.
http://en.wikipedia.org/wiki/Well-Tempered_Clavier

Eventually the equal tempered system was adopted universally in western cultures, but even down to the 19th century, some keyboard instrumentalists refused to give up the purity of the perfect fifths and persisted with the old systems of tuning.

In concert performances which do not use fixed tuned instruments, deviations from equal temperament are common. As will be explained later, the context of certain harmonies requires a differentiation for example, between an F# and a Gb. Also, certain styles of music, like the choral works of the Renaissance, demand a purity for the intervals that is unavailable in equal temperament.

Melodic Analysis

Now to return to more familiar ground, the reader is encouraged to sing the "Do-Re-Mi" song a couple of times to fix its melodic contours in mind. Up to this point we have been discussing one of the variables of music, namely pitch. This refers to the actual frequencies of tones used in a composition and, as we have seen, these are organized into scale patterns. The principal tones of the song are in the scale of C major, with certain other tones from the chromatic scale used incidentally. This appearance is dictated by harmonic considerations that will be discussed later. To visualize the melodic patterns, the last repetition of the refrain of the song has been graphed in Figure 4, and coordinated with the words and music.

The graph has a vertical axis, graduated like the rulers, in units representing a complete octave of the chromatic scale, from C to c. Fortuitously, the tones used in the melody are encompassed within this span. The horizontal axis represents another variable, that of duration. The vertical lines divide the refrain into 32 measures with four additional measures (nos. 33-36) ending the song with a rising scale intoning the musical syllables. The duration of each note in the song is shown by the shaded blocks at the frequency levels of the tones sung.

Figure 4

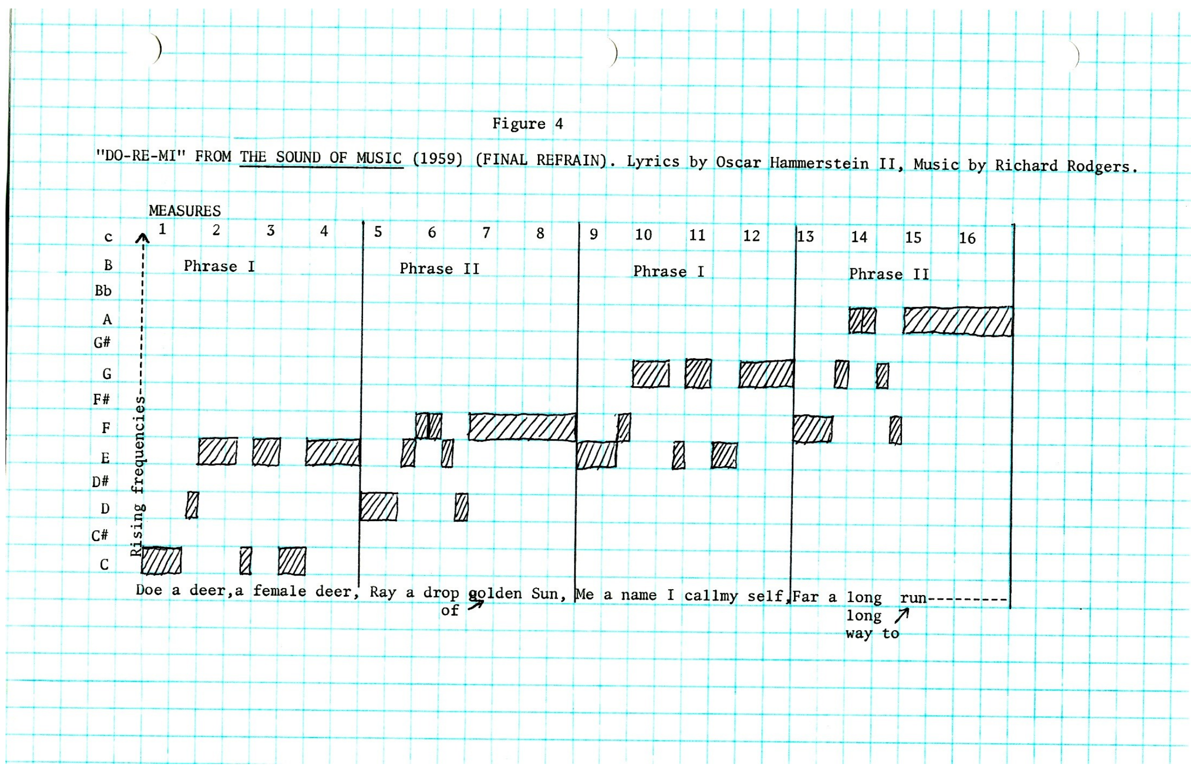
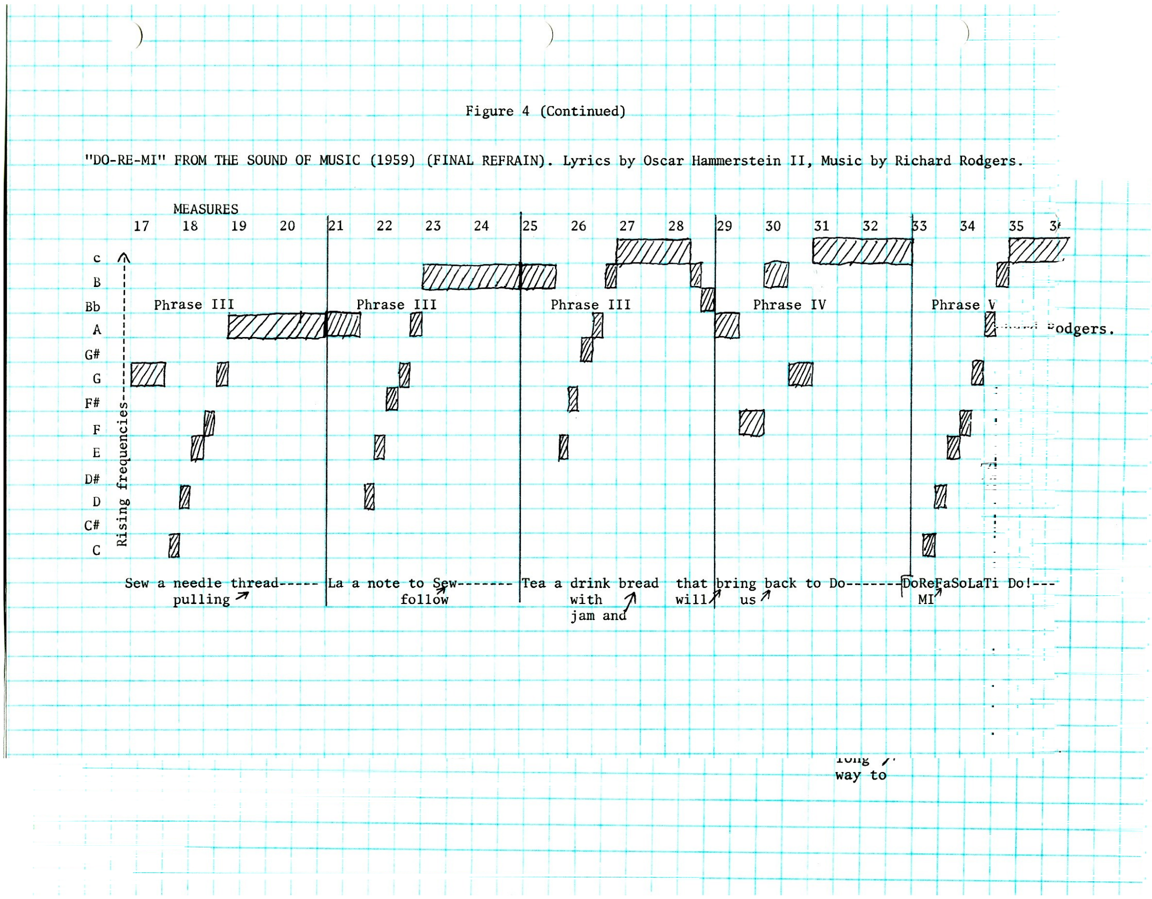


Figure 4
"DO-RE-MI" FROM THE SOUND OF MUSIC (1959) (FINAL REFRAIN). Lyrics by Oscar Hammerstein II, Music by Richard Rodgers.
MEASURES
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Phrase I
Phrase II
Phrase I
Phrase II
c
B
Bb
A
G#
G
F#
F
E
D#
D
C#
C
Rising frequencies
Doe a deer,a female deer, Ray a drop of golden Sun, Me a name I callmy self, Far a long long way to run---------

Figure 4 (continued)



Phrase Structure

The graph reveals the structure of the melody. Each four measures coincides with a line of the lyrics and is musically called a "phrase." Identical phrase structures are found in measures 1-4 and 9-12 (Phrase I); measures 5-8 and 13-16 (Phrase II); measures 17-20, 21-24, and 25-28 (Phrase III). The refrain is essentially concluded with Phrase IV and the entire song with Phrase V.

The refrain is divided up into two main sections; the first from measures 1 to 16, and the second from measures 17 to 32. In melodic terminology these are called the "antecedent" and the "consequent." In this song, both the antecedent and consequent exhibit an overall rising curve, in keeping with the subject of the song, the rising scale sequence. Rising melodic lines have a tendency to build up tension. When this is combined with a pattern of notes of shorter duration as in Phrase III, the tension building is accelerated. This tension is finally resolved in Phrase IV. The chromatic accidentals that make their appearance in measures 22, 26, and 28 are a result of the sequential form of Phrase III and its harmonic underpinning. We shall return to these accidentals in the later discussion of harmony.

Tempo

There are several points to be made about the duration variable. First of all, the refrain is prefaced by the notated comment "in spirited tempo." Tempo is the rate of speed in which the song is sung. In the cast recording of The Sound of Music, this was clocked at 132 beats per minute. In each measure of the song there are two beats, indicated in the graph by the separate squares. An accurate instrument for indicating tempo is the metronome invented by Winkel and first patented by Maelzel in 1816. http://en.wikipedia.org/wiki/Metronome

The first composer to use the metronome was Beethoven. Most composers tend to use suggestive words rather than strict metronome indications to guide the performer in selecting tempi. In musical performance it is rare to hear a strict observance of the tempo throughout a piece. Human beings seldom function as machines and this is probably one of the reasons that musicians will never be replaced by computers or other modern means of producing musical sounds. Intelligent and expressive deviations from tempo are part of the elusive human variables of music that will never be accessible to mathematical analysis.

Meter and Rhythm

It will be noticed in the graph that most of the notes are not equal in duration to a single beat. The first note, for example, is 1.5 times the duration of a single beat and this is followed by a note that is 1/2 the beat's duration. This sequence of longer and shorter imparts the peculiar lilting quality to the melody. The rapid burst of short notes in Phrase III is also effective in suggesting the rising scale theme of the whole song. The main mathematical consideration is that the duration of all the notes are proportionally related to the main beat. The durations are all integral multiples of 1/2; i. e., 1/2, 2/2, 3/2, and 4/2.

The measures organize the beats into accent patterns. The first beat of each measure receives an extra stress, hence the terminology "downbeat." The second beat is unaccented and is sometimes called the "upbeat." This cyclical pattern of accent is sometimes suspended for structural reasons. For example, the extension of the tone F in measure 8 neutralizes the downbeat. This has the effect of building the expectation for the strong reentry of Phrase I in measure 9. The same structural device is used to excellent effect in the repetitions of Phrase III.

The arrangement of beats into measures is known as meter.
In "Do- Re-Mi" the basic beat is designated musically by a "quarter" note. Since there are two beats per measure, the meter, two-four, is notated like a fraction without a bar line separating the 2 and 4. The song, "My Favorite Things," from the same musical is in three-four meter and it moves like a typical waltz with three beats to a measure. Meter provides a skeletal structure for creating rhythms, probably the most difficult variable to describe. The rhythmical patterns of a waltz are regular and cyclical, necessitated by the periodic motion of the dance with which it is associated. Most of the

music we enjoy has this quality of rhythmic continuity and sense of beat. In larger works of the symphonic repertoire, however, contrast is achieved by alternating passages with different rhythmic structures.

So far, the analysis of "Do-Re-Mi" has brought out the essential elements that give the melody its unique identity: tones arranged into phrases, phrases which relate to each other in patterns of repetition (i.e., the phrase sequence I, II, 1, II, III, III, III, IV), an overall directional flow (in this case a rising line determined by the scale theme), and a consistent cyclical rhythmic pulse.

Harmony

Rhythm and melody are the basic foundations of music in all cultures. In Western civilization, however, there is another dimension, that of harmony, which distinguishes its music from all others. The use of the word "dimension" is purposeful, for harmony has many features in common with perspective in the visual arts. Essentially it creates a relationship between tones that imparts an aural sense of mass and depth. Normally music students pursue at least one year studying harmony. The description here will necessarily be cursory, focusing on its basic theoretical foundations in keeping with the objective of revealing mathematical content.

Recall that the octave is the simplest interval relating two different tones and that it functions most importantly in constructing scales and allowing transposition of melodies into different registers. The first interval with harmonic meaning is the perfect fifth, its ratio of 3:2 being the next simplest after the octave's 2:1 ratio.

Referring to the sequence of fifths derived in Chapter 1, F,C,G,D,A, E,B, remember that this is really a geometric sequence with a common ratio of 3/2. Theoretically, we could extend the sequence in both directions forever. As soon as the sequence is extended beyond the seven notes shown, other notes appear with accidentals:

... ,Fb,Cb,Gb,Db,Ag,Eb,Bb, F,C,G,D,A,E,B, F#,CI,G#,D#,A#,E#,B#, ...

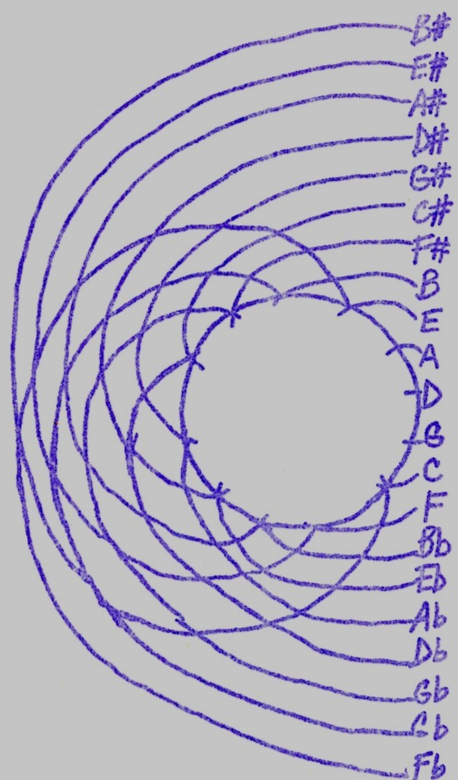
that are in the same alphabetical pattern. Beyond B# the sequence contains the same alphabetical pattern in double sharps. Likewise, below Fb, double flats are introduced. Double sharps and flats occur less frequently in musical notation and only if they are critical to the composer's harmonic intentions. Triple sharps and flats are never used in composition.

Enharmonic Tones

With equal temperament, the open-ended sequence above is converted into a closed sequence that is analogous to the "Wrapping function" familiar to students in trigonometry. That is, a many-to-one correspondence is established between the tones represented by the sequence of fifths and the twelve steps of the chromatic scale. Figure 5 shows this correspondence.

Figure 5

Figure 5



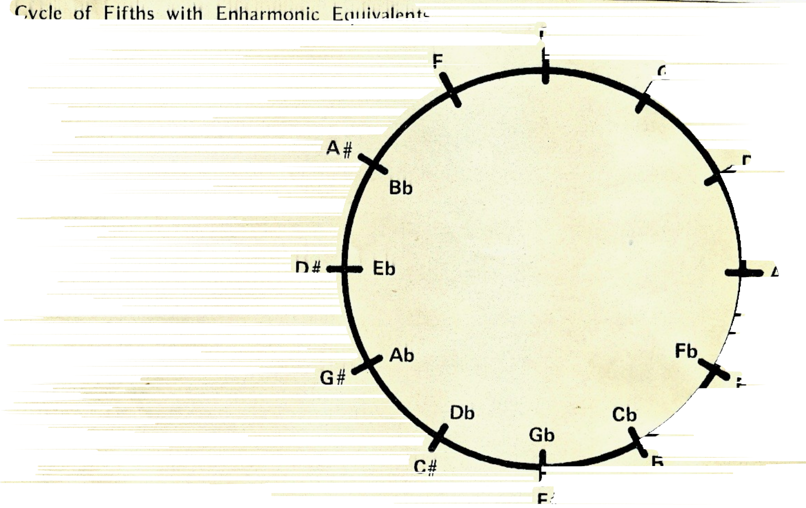
FORMATION OF THE CYCLE OF FIFTHS
FROM THE SEQUENCE OF FIFTHS
BY MEANS OF A WRAPPING FUNCTION.
THE CREATION OF ENHARMONIC EQUIVALENCE
IN THE TEMPERED CHROMATIC SCALE.

The resulting structure is known as "the cycle of fifths." It establishes the pattern of enharmonic equivalents mentioned earlier. The mapping shows this equivalence by the curves spiraling in from tones onto the circle at points of coincidence. This is the basis for assigning the same key on the piano to F# and Gb, C# and Db, and so forth. (Figure 5A).

The principles of functional harmony govern the movements from one chord to another. A chord is a set of simultaneously sounding tones. Harmony is functional when a logical structure is perceptible underlying the progression of chords. The most common movement is dictated by the cycle of fifths. This is the essential movement apparent in the harmonic structure of "Do-Re-Mi."

Figure 5A

Cycle of Fifths with Enharmonic Equivalents



Root Progression

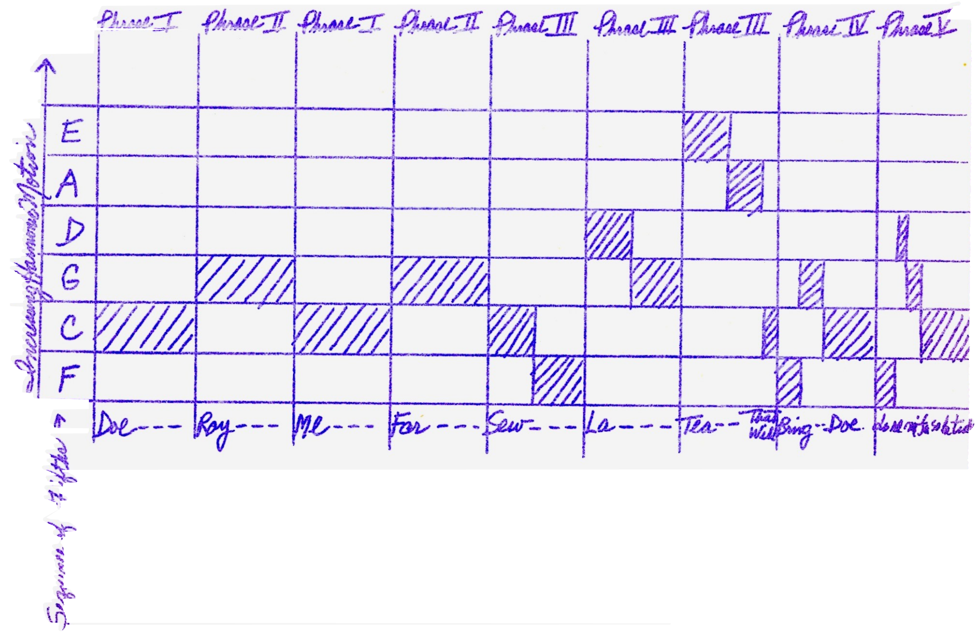
To understand this principle it is important to understand that chords are structured on foundation tones that are called "roots. " We shall speak of a C chord if the tone C is the root of the chord. Similarly, an E chord has E as its root, etc. The basic progression of roots is shown for each line of text as follows:

Text Line:	Doe a deer, a female deer,
Root Progression:	C------------------------------
Text Line:	Ray a drop of golden sun,
Root Progression:	G-----------------------------
Text Line:	Me a name I call myself,
Root Progression:	C-----------------------------
Text Line:	Far a long, long way to run,
Root Progression:	G-------------------------------
Text Line:	Sew a needle pulling thread,
Root Progression:	C-----------------------------F
Text Line:	La a note to follow sew,
Root Progression:	D-----------------------G
Text Line:	Tea a drink with jam and bread
Root Progression:	E---------------------------------A
Text Line:	That will bring us back to doe!
Root Progression:	C-----------F---------G--------C
Text Line:	Do-re-mi-fa-so-la-ti-do!
Root Progression,	F-------D-------G-------C

The root progression is illuminating when positioned on a graph (Figure 6). It shows that harmonic motion differs from the scale pattern of melodic motion. The vertical axis will now consist of the sequence of fifths, restricted to the actual roots that appear in the song. The horizontal axis has been compressed so that one square is equivalent to one measure. The first thing to notice is the changing harmonic rhythm. The first half of the song, from "Doe" to "Far" has one root per phrase. When Phrase III enters, the rate of change of roots doubles. Then at the end of the third repetition of Phrase III with the line beginning with "Tea," the rate changes to three times what it was at the opening of the song, coinciding with the climax of the refrain. The acceleration of harmonic change generally contributes to the building up of tension.

FIGURE 6

HARMONIC ROOT PROGRESSION for "DO-RE-M



Tonic and Dominant

In the key of C major, the root C is home base. It is given the name "Tonic" and it is the harmonic goal of resolution. After the climax is reached at the end of Phrase III on the root A, Phrase IV takes us back to C. In music we call Phrase IV a "cadence." The purpose of a cadence is to clearly reach some harmonic goal. The Tonic root is the natural place to end a song giving it a sense of completion. Phrase V is of course an afterthought, a kind of finishing touch, which of course still resolves on the Tonic root, C.

It is also common to begin a song by defining the Tonic root as it is done here. This sets up a level of equilibrium to which all other roots relate. The Tonic functions as a sun surrounded by the other roots as satellites.

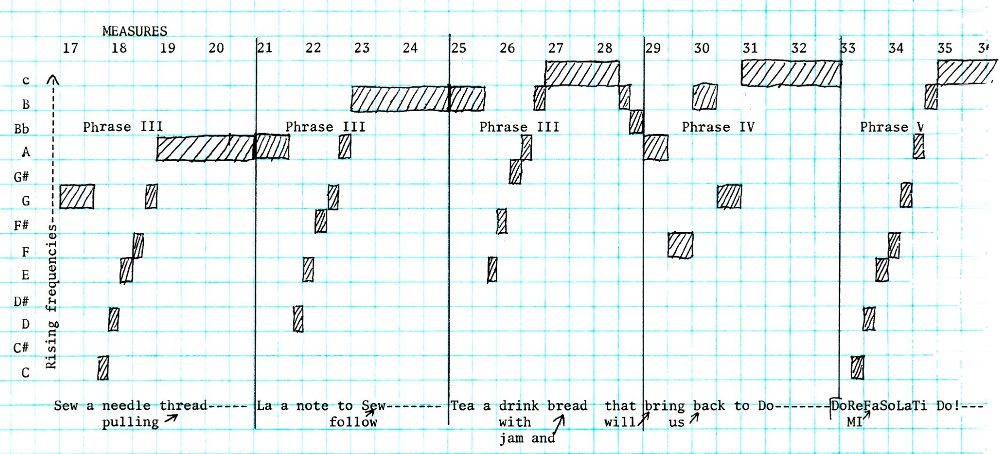
The root most closely related to the Tonic is its fifth, in this case, G. The movement from C to G is an increase of potential harmonic energy. This is what occurs in the repeated sequence of Phrase I followed by Phrase II. This pattern at the beginning of the song functions to overcome the inertia of the stable Tonic. The root G functions in musical terms as a "Dominant." The Dominant-Tonic relationship is the strongest and most incisive kind of harmonic movement. Moving from the Dominant to the Tonic produces the strongest resolution and this is what occurs at the end of Phrases IV and V. It has the analogous effect of converting potential energy into kinetic energy that is encountered when an object falls to earth from a given height. The increasing harmonic tension which occurs in the repetitions of Phrase III makes excellent use of the Dominant to Tonic movement by setting up what are called "Secondary Dominants". These are transitory modulations that momentarily permit other roots to assume a Tonic role.

Thus, at the first appearance of Phrase III, the root C undergoes a chord structure change which converts it into a Dominant. The movement to the root F sounds to our ears then as a bona fide Dominant to Tonic resolution. However, this is quickly followed by the jump to the higher potential energy of the root D and the same pattern is repeated as a kinetic Dominant to Tonic resolution to G. Finally this secondary Dominant to transitory Tonic pattern is climaxed with the progression from E to A, through the cycle of fifths. It's no coincidence that the climax of the song coincides with the highest level of harmonic tension in the song. These brief modulatory episodes take the musical flow to keys other than C major and as a result, tones from these foreign scales make their appearance in the melody. This is the explanation for the logical appearance of the tones F# and G# in the melody during the repetitions of Phrase III (Figure 4).

The harmonic movement throughout this song is governed by the sequence of fifths. Movement in the direction of increasing harmonic tension is balanced by movement in the direction of resolution. This is the way harmony "breathes" and it is the composer's job to see that the harmonic movement coordinates with the melodic flow. It's one of the principles of craftsmanship for music composed in the style of functional harmony.

Figure 4 (Continued)

"DO-RE-MI" FROM THE SOUND OF MUSIC (1959) (FINAL REFRAIN). Lyrics by Oscar Hammerstein II, Music by Richard Rodgers.



The Subdominant

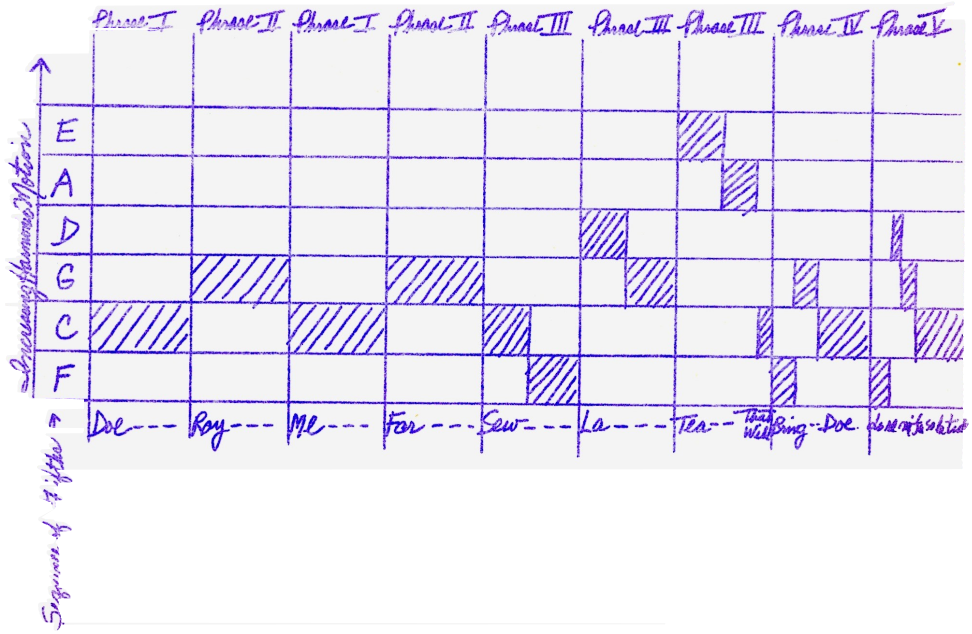
The sequence of fifths contains another principle of harmonic movement which completes a kind of trinity for root relationships. This principle may be described as the "Subdominant" connection. It can be seen in (Figure 6) that the root F lies directly below the Tonic C in the scale of harmonic tension. F is the Subdominant of C. Its principal function is to reinforce the directional flow from the Dominant to the Tonic. This can be seen in the cadences graphed for Phrases IV and V. First, in Phrase IV the jump from F to G inputs energy into the system that requires resolution to the Tonic. This is the most common cadential formula in tonal music (i.e., music based on the principles of tonality).

In Phrase V, this is accentuated even further by the increase in tension from the root F to the root D, requiring a two-step Dominant to Tonic resolution, D to G; and G to C. Movement to the Subdominant always implies movement away from the Tonic. If it is followed by movement to the Dominant, the pulling effect of the Tonic is immediately felt. These forces are the musical analogues of centripetal and centrifugal forces that are familiar to us in the models of planetary motion. It's no accident that the discovery of these principles in music occurred at the same time as the presentation of Newton's Law of Gravitation.

On the other hand, the energy jumps implied by the movement from one root to another is more closely akin to the model of the atom illuminated by quantum theory. In this analogy, the basic quantum of energy in harmonic motion is the perfect fifth and the harmonic spectrum is the sequence of fifths. The Tonic functions in a way analogous to the nucleus of

FIGURE 6

HARMONIC ROOT PROGRESSION for "DO-RE-M



an atom, the other roots behaving like electrons held in orbits by the attractive force of the Tonic nucleus. Analogies are, at best, approximations to getting at the truth of the matter. However, at the beginning of the Harvard Dictionary of Music, there is a quotation from the Talmud which is particularly appropriate for this discussion: "If you want to understand the invisible, look carefully at the visible." [2]

Texture

The system of connecting chords by root relationships that we have been discussing is known as tonality. It evolved in the 17th century as an extension of Pythagorean theory and was first codified by the French composer and theorist, Rameau, in his Traite de L'Harmonie of 1722. It came to dominate musical practice after a long and gradual evolvement from polyphonic music. Polyphony is a style of music that stresses the independence of melodic lines heard simultaneously. This style reached its culmination in the High Renaissance with the works of Josquin des Pres. The scales used at this time were called modes and were derived from the Gregorian chant music of the Church. Although certain tones in the modal scales were considered more important than others, the relationships were purely melodic. In this kind of music, chords occur as fall-out, so to speak, of the independent melodies that were heard simultaneously. The Baroque period that followed explored the harmonic results produced by this counterpoint for their expressive effects and evolved tonality as a consequence. The most perfect fusion between counterpoint and functional harmony occurred at the end of the Baroque period in the music of Johann Sebastian Bach. By this time, modality had been abandoned to be replaced by the major and minor scales which are fundamental to the system of functional harmony. The composers who followed Bach, including his own sons, adopted the new harmonic style, emphasizing the composition of a single melody supported by an expressive harmonic foundation. Tonality dominated musical styles down to the end of the 19th century and it still persists with considerable vigor in the popular music of our times.

Once the principles of tonality had been mastered in the 18th century, composers found that it was adaptable to contrapuntal techniques. The works of Haydn and Mozart display a new kind of elegance in the use of counterpoint. The inner voices began to take on a new life of their own and even the bass, formerly restricted to the role of supporting the harmonic flow, frequently becomes melodically significant in their music. The contrast between polyphony and homophony emerged at this time as a new variable in music, that which can best be described by the word, texture. In works of an extended character chordal passages are contrasted with those of a contrapuntal nature and sometimes they are combined into an intricate weave.

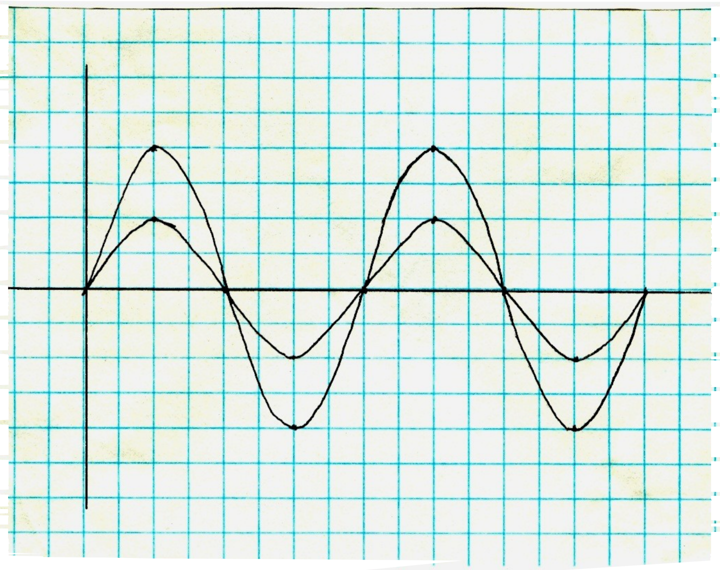
The 19th century witnessed the invention and perfection of orchestral instruments that were well suited to emphasizing contrasts in texture through exploitation of their different tone qualities. The study of orchestral textures is a fundamental part of the science of orchestration. It depends upon an intricate knowledge of instrumental construction and capabilities and their effects when combined. Many of these relationships are expressible in mathematical terms. Tone quality, for example, is a direct result of the mathematical partials (harmonics) which enter into the formation of tones.

Dynamics

The last variable to be discussed is that of dynamics. This is a general term which includes all of the elements contributing to the degree of loudness (or softness) in a musical passage. When a tone is sounded its periodic vibrations can be pictured as a composition of sine waves (Figure 7). The resultant curve fluctuates about a mid-line of equilibrium, the peaks and troughs giving a measure of energy put into the vibrating system. This is called the amplitude of the sound. It registers in our minds as a certain level of loudness. Since our hearing is most sensitive to sounds which lie in the highest octave of the piano, it takes much less energy to produce audible sounds in that range. This is why a single piccolo can be heard predominating over the entire orchestra in full blast. Mathematically, loudness is measured on a subjective scale based upon a unit called the bel, named in honor of the inventor of the telephone, Alexander Graham Bell. One tenth of this unit, the decibel, is the smallest change in loudness that the ear can detect.

Figure [illegible]

Sine wave analysis of two tones having the same frequency but different amplitudes. The tone that diverges farther from the axis of equilibrium has twice the intensity of the other tone.



The sounds heard in music vary from about 25 decibels (the softest violin tone) to about 100 decibels (the strongest sound of the full orchestra). Loudness varies with the logarithm of intensity. This means that 100 violins can produce only twice as much volume as 10 violins. This places a natural limitation on the size of an orchestra. An orchestra performing the symphonies of Mahler and Bruckner is about the maximum size needed to produce the massive sounds they composed. Beyond this magnitude, the logarithmic scale indicates diminishing returns in producing sheer volume.

Texture is commonly joined with dynamics to produce contrast in music. The alternation between solo instrumental sounds and ensemble playing is a technique for producing this contrast. Dynamic contrast can also be used on a large scale to structure a composition. For example, the entire second movement of Beethoven's Fourth Piano Concerto is a dialogue between the solo instrument and the orchestra with dynamic contrast being the predominant structural element of the drama. (S13)

http://www.culturalmath.com/media/Sound-13.mp3

In most of the popular music we hear, a constant level of dynamics is maintained. Symphonic music, on the other hand, employs dynamic contrast in building up its larger forms. Indeed, one of the reasons why many people find it difficult to appreciate symphonic music is the fact that they have been nurtured on music which has a steady level of dynamics. The frequent changes in dynamic levels occurring in symphonic music requires a different orientation that can be disconcerting in the uninitiated.

As an ensemble piece, "Do-Re-Mi" is structured on a steadily rising dynamic curve, called a crescendo, as the children become more vigorously involved in the successive repetitions of the refrain. The buoyant enthusiasm of the composition is a direct result of this progression to a dynamic climax.

Time--The General Variable

All of the variables of music are bound to the dimension of time. From an abstract point of view, time may be considered the most general variable of music, functioning in an all-pervading way as space does in the visual arts. In science, time is the independent variable for the analysis of motion, and, as we have seen, music is a complex ordering of tonal motion.

Time in science is an objective quantity; in music it is subjective. What this means is that the time-structured events of music come close to replicating the structure of time as it is humanly experienced. Our lives are structured into a progression of time blocks of varying levels of significance. Any two periods of our existence may have the same time dimension but radically different intensities. Music shares this structural characteristic. As a result, it is the one art that comes closest to expressing the forms of human emotions. This is the reason why music has the power to evoke the deepest response. We shall see how this is accomplished through the marshaling of all of its resources in the next chapter.

LIST OF RECORDINGS

S12

RODGERS & HAMMERSTEIN: THE SOUND OF MUSIC: DO-RE-MI SONG FROM THE ORIGINAL LP RECORDING, NOW AVAILABLE ON THE 50TH ANNIVERSARY CD AND MP3 EDITION.

http://www.amazon.com/gp/product/images/B002TYGJEQ/ref=dp_image_text_0?ie=UTF8&n=163856011&s=dmusic

S13

BEETHOVEN : PIANO CONCERTO NO. 4, LEON FLEISHER, PIANIST, CLEVELAND ORCHESTRA, GEORGE SZELL, CONDUCTOR, FROM THE ORIGINAL COLUMBIA SET OF ALL FIVE CONCERTOS. COLUMBIA SET MX 30052.