

# THE CULTURAL IMPACT OF MATHEMATICS

## UNIT I

### Chapter 3 - The Melancholia I by Albrecht Durer

In startling contrast to the buoyant enthusiasm and self-confidence in man portrayed in Raphael's School of Athens, we shall move to the North and examine an engraving by the introspective German artist, Albrecht Durer (1471-1528). [http://en.wikipedia.org/wiki/File:D%C3%BCrer\\_Melancholia\\_I.jpg](http://en.wikipedia.org/wiki/File:D%C3%BCrer_Melancholia_I.jpg)

Kenneth Clark gives a most perceptive analysis of this scene:

"Durer," he says, "was deeply engaged by the mystery of the human psyche. His obsession with his own personality was part of a passionate interest in psychology in general, and this led him to produce one of the great prophetic documents of western man, the engraving he entitled Melancholia I. In the Middle Ages melancholia meant a simple combination of sloth, boredom and despondency that must have been common in illiterate society. But Durer's application is far from simple. This figure is humanity at its most evolved, with wings to carry her upwards. She sits in the attitude of Rodin's Penseur, and still holds in her hands the compasses, symbols of measurement by which science will conquer the world. Around her are all the emblems of constructive action: a saw, a plane, pincers, scales, a hammer, a melting pot, and two elements in solid geometry, a polyhedron and sphere. Yet all these aids to construction are discarded and she sits there brooding on the futility of human effort. Her obsessive stare reflects some deep psychic disturbance. The German mind that produced Durer and the Reformation also produced psychoanalysis. I began by mentioning the enemies of civilization: well, here in Durer's prophetic vision, is one more way in which it can be destroyed, from within. "1

Another interpretation, more personal but equally valid, is that the engraving represents Durer's creative Muse rendered despondent by the recent death of his mother. Certainly one who has suffered a great loss has an intimate knowledge of the paralysis of depression. Yet, as this engraving testifies, Durer was able to transcend his grief and produce a work that universally expresses this emotional state. The true artist can recover his soul in the process of creation. There is no question but the *Melancholia* reflects Durer in complete control of his genius. Absorbed in his work, the artist is restored to emotional health. Indeed, as further analysis will show, there is a rich element of therapeutic play in this composition. It comes in the form of magic, a peculiar susceptibility of the medieval mind.

A hint of this is given by Durer's original magic square beneath the bell (Figure 1).

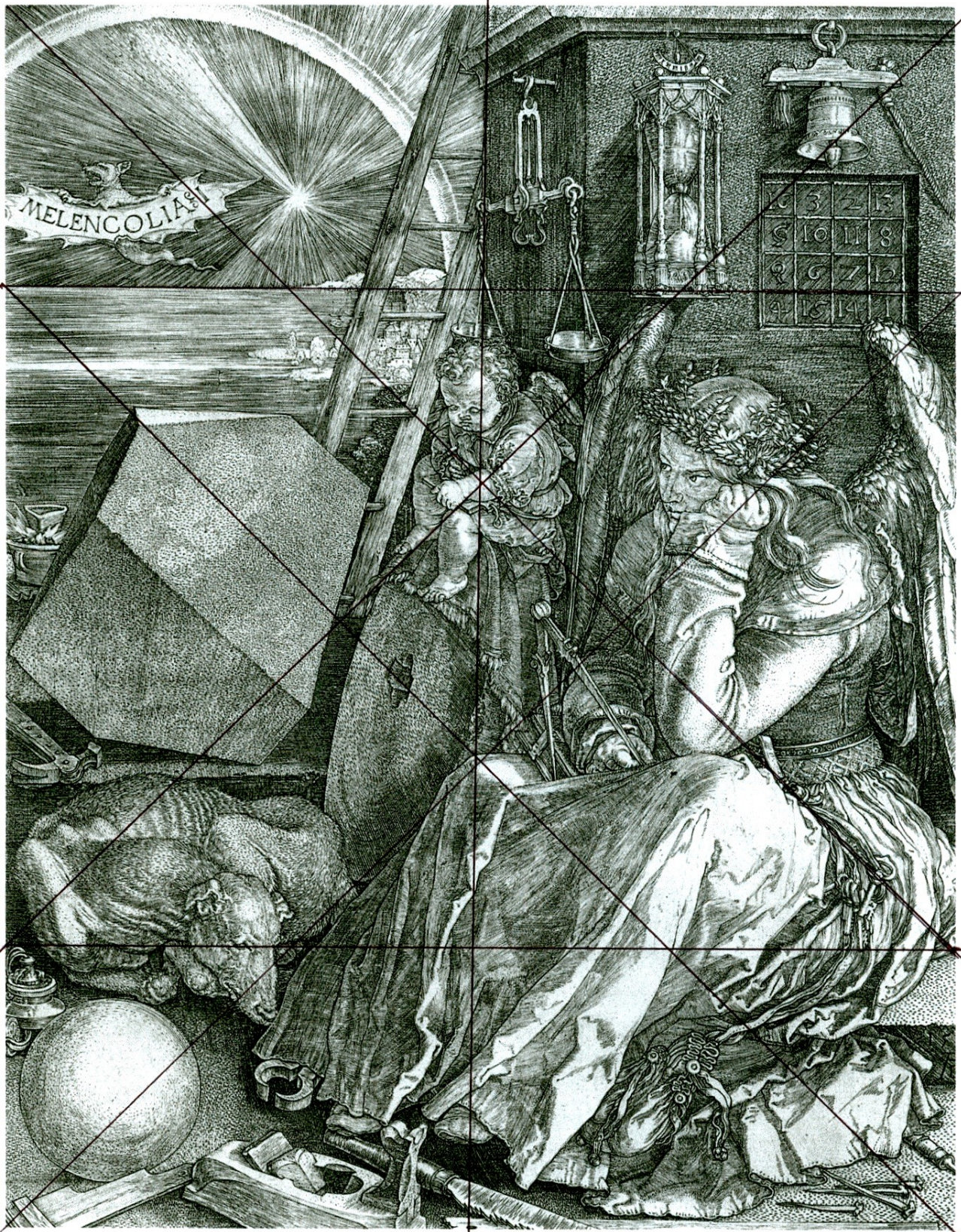
# Dürer's Magic Square from the Melancholia I

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

In the Middle Ages, magic squares were often worn as amulets to ward off evil spirits, an appropriate creation for one who has been so closely touched by death. The magic of this array of numbers lies in the fact that when the numbers along any row or column or diagonal are added, they all yield the constant sum of 34. It may also be noticed that the central square of numbers and the corner squares all yield the same sum. Durer adds a delightful touch to its construction by providing us with the year of the engraving, 1514, these two numbers being adjoined in the bottom row. In addition, the ratio of the length to the width of the engraving is approximately 16:13, determined by the numbers in the top corners. We shall now see how this proportion is used in the framework of the composition.

From the bottom and top, the width is folded over to produce two overlapping squares, what Bouleau<sup>2</sup> calls the "rabatment" of the shorter side on to the longer side. The first important contribution of this construction is to define the horizon line. It is the upper horizontal which terminates on the left in a point which serves as the principal vanishing point (V.P.) for the perspective organization (Figure 2). On the side of the building the horizon line defines the bottom of the hour glass and tops the lower row of numbers in the magic square before terminating at the right edge.







A matrix of oblique lines is automatically determined by the rabatment. The intersections of the diagonals of the upper and lower squares form a central diamond. A side of the diamond is equal to the length of the diagonal of the magic square, and this length is also used to give the proper positioning of the magic square measured from the right edge of the engraving.

Each of the oblique lines contributes to the positioning of compositional details. For example, the line from the upper right corner to the left edge of the horizontal above the sphere,

- 1) sets the dimensions of the bell-holder and the bell;
- 2) cuts across the diagonal of the small square containing the numeral '16';
- 3) intersects the horizon line at the base of the central axis of the hour glass;
- 4) defines the tip of the woman's left wing;
- 5) orients the cherub's body by connecting its shoulder, wrist, and knee;
- 6) frames the backward extension of the wheel on which the cherub sits; and
- 7) finally takes note of the intersection of the front haunch and neck of the sleeping dog before terminating at what appears to be a tankard.

Notice also how the waist-lines of the cherub and woman are coordinated by the oblique line from the upper left corner to the right edge of the lower horizontal. The line drawn from the lower left hand corner is important in connecting key details of the woman's figure. As we shall see later, the rabatment framework is also the key to organizing the perspective of this work.

The central axis of the engraving is important for defining the left edge of the building. After this analysis, one could add to the many interpretations of Durer's profound puzzle that the figure in the engraving is pondering the mysteries of her own creation. One can almost see a playful smile about her lips, perhaps in anticipation of another breakthrough in self-discovery.

The foregoing suggestion is not too far-fetched. Clark's interpretation stops short of probing for the "deep psychic disturbance." This is frustrating for the

viewer trying to make sense out of the contradictions apparent in the scene. On the one hand there are the tools and measuring implements of the builder strewn around in disarray. On the other hand, the mal-formed dodecahedron, the magic square, and a melting pot; the latter conjuring up the alchemist's art of trying to extract gold from baser metals. There is a conflict in all of this, one that dates back to the time of Pythagoras and still persists in the world of today. It is the age-old struggle between the mystical and rational manifestations of the human mind. No other idea excites this conflict more keenly than the Pythagorean conception of the universe.

On the one hand, this conception gave rise to the science of mathematics, the most rational of man's creations; on the other hand, it nurtured number mysticism, the most tempting foundation for the mind's contemplation of the irrational.

Durer knew this conflict well and struggled with it more than any other man of his time. As a Renaissance man he stood at the threshold of the historical separation between faith and reason. Two hundred years earlier, William of Occam (c. 1290-1349), announced this separation and was eventually excommunicated for his ideas. [http://en.wikipedia.org/wiki/William\\_of\\_Ockham](http://en.wikipedia.org/wiki/William_of_Ockham) The medieval world had to fold before the spirit of renaissance could provide the conditions to free "logic from metaphysical and theological encumbrances."<sup>3</sup>

Durer personified this transition in his art. Like da Vinci, he was in the forefront of those who approached art with scientific precision. His writings on perspective recorded the developments that had originated in Italy and they had a widespread influence on the artists of northern Europe.

[http://en.wikipedia.org/wiki/Linear\\_perspective](http://en.wikipedia.org/wiki/Linear_perspective)

In addition, his treatise, A Course in the Art of Measurement with Compass and Ruler, was the first practical guide to the mathematics of Ancient Greece to be published in the North. It covered constructions of algebraic curves like the conic sections, regular polygons, the Platonic solids and spiral forms. It

was not only a source book for artisans, but it also provided stimulation to the mathematicians and scientists of the later 16th and 17th centuries; men like Cardano, Galileo, and Kepler. <http://en.wikipedia.org/wiki/Durer>

Durer's mind ranged over a vast array of engineering problems, from city-planning to the design of sun-dials. He perfected the various media of engraving, woodcut, and etching in addition to exploring the subtleties of oil painting. He shared with Leonardo a pragmatic interest in Nature, carefully observing and recording its details. He revised and extended the theory of human proportions so that in his own time “... on both sides of the Alps, art had become a matter of direct and personal contact between man and the visible world.” <sup>4</sup>

[http://en.wikipedia.org/wiki/File:Durer\\_Young\\_Hare.jpg](http://en.wikipedia.org/wiki/File:Durer_Young_Hare.jpg)

[http://en.wikipedia.org/wiki/Great\\_Piece\\_of\\_Turf](http://en.wikipedia.org/wiki/Great_Piece_of_Turf)

To help in understanding his mastery of perspective, (Figure 3) shows how the rabatment framework was utilized in determining key lines of perspective. As mentioned earlier, the top horizontal of the lower square places the horizon line. Durer plays with our sense of perspective by using the Sun as a “false” vanishing point. It appears, for example, that the top cornice of the building is foreshortened in the direction of the Sun. The balance arm, however, sets things aright again by correcting the focus to the real vanishing point on the left edge of the horizon line.

When lines of perspective are drawn from the vanishing point through the vertices of the central diamond, an interesting fact of composition is revealed. The three top vertices of the pentagon facing the woman are contained on these lines. If, in addition, perspective lines are connected to the intersections of the lower horizontal with the oblique line from the lower left corner and also to the central axis, the remaining vertices of this pentagon are also included. In other words, it would seem that Durer has made use of the polyhedron as a sighting instrument for controlling his perspective organization.







This may be one explanation for the peculiar shape of the polyhedron and its dominant position as part of the composition.

Additional lines of perspective have been drawn in to show the proportioning that occurs between the figures of the woman and the cherub. For example, their eye levels are joined through a single line to the vanishing point. One might conjecture whether this is the object of her obsessive stare. Finally, it is worth noting that the center of the Sun is positioned on the diagonal of the rectangle above the horizon line.

With all of his involvement in the scientific rendering of compositional detail, Durer was still a man of his times when it came to the content of his works. Look, for example, at his self-portrait.

[http://en.wikipedia.org/wiki/File:Durer\\_self\\_portrait\\_28.jpg](http://en.wikipedia.org/wiki/File:Durer_self_portrait_28.jpg)

What a remarkable self-study it is! If one did not know it was Durer, it could easily be mistaken for a portrait of Christ.

The religious element is deepened by a sense of mystery. For although Durer was a superb craftsman and technician, his art conveys the introspection of a spiritual man. He reveals this aspect of his personality most dramatically in the Gothic mysticism of such prints as the Four Horsemen of the Apocalypse.

[http://en.wikipedia.org/wiki/File:Durer\\_Revelation\\_Four\\_Riders.jpg](http://en.wikipedia.org/wiki/File:Durer_Revelation_Four_Riders.jpg)

All of his rational thought is placed at the disposal of medieval concepts of allegory and symbolism. It is in looking at these mystical works that we come full circle to the dilemma facing the pensive figure in the *Melancholia*. Durer's personal struggle is the same which perplexes modern man.

The tools of science are near at hand. They are available to further our knowledge of reality and to give us control over nature. Yet, what shall be lost in the process? Must man's enjoyment of the fruits of knowledge always be joined by a wider separation from his spiritual beliefs?

Durer and the Humanists of the 16th century stood at the crossroads of

metaphysical thought. The Pythagorean tradition, with its total integration of scientific reasoning and mystical faith, had been revived and absorbed by the great minds of the Renaissance. As Galileo was to say, "Mathematics is the language in which God has written the universe."

Others were to take up the tools symbolized in the Melancholia and reach out to "discover" the language of the "Supreme Mathematician". Eventually, however, it would lead to a complete repudiation of the existence of God.

At the beginning of the 19th century, Pierre Simon Laplace published his great work on celestial mechanics completing the mathematical perfection of the Newtonian universe. He was asked by Napoleon why there was no mention of God in his work. His answer fulfilled Durer's troubled vision. "Sire," he said, "I have no need of this hypothesis."

<http://en.wikipedia.org/wiki/Laplace>

Taken together, The School of Athens and the Melancholia reflect the hopes and fears of our modern existence. The philosophical questions that they posed are the same that we are wrestling with today. Science and theology have long since gone their separate ways as Occam recommended, each compelled by practical necessity to stake out their own domains. Yet, on occasion they are forced to meet face to face at the outer limits of theoretical science. Contrary to popular opinion, science is not a plodding compilation of "hard, irreducible facts." At the creative level, it requires the same incredible balance between intuition and rational thought that is to be found in a great work of art. In a series of lectures entitled, Albert Einstein and the Cosmic World Order, the distinguished mathematician, Cornelius Lanczos describes Einstein's discovery of the Theory of Relativity: "From .•• very general speculations he suddenly arrived with logical necessity at his equations. This was not according to the established rules of the game. This was black magic more than anything else. To stare in empty air and pullout results from nothing as a result of speculations, as

the old Greek philosophers were wont to do, was disdained since the time of Galileo as a nonsensical procedure. One had to experiment first and see what happened. Then one tried to codify these experiments by a mathematical equation. This had been the well-established rule for hundreds of years. And now Einstein dared to challenge this procedure by reverting to the dreams of the ancients who tried to understand nature on the basis of logical deductions rather than describe it on the basis of carefully conducted experiments. The magical thing about Einstein was, however, that he succeeded where the ancients failed. He had the mighty tools of mathematics at his disposal, developed in a slow evolution of more than two thousand years since the time of Plato and Aristotle. The equations of Einstein, arrived at by purely speculative means, told us that indeed the planets had to move around the sun in ellipses and not in something else because our universe is not any mathematical universe, but a marvelously reasonable and well-ordered mathematical universe, pervaded by a supreme Cosmic Wisdom." **5**

<http://en.wikipedia.org/wiki/Einstein>



## LIST OF REFERENCES

### Chapter 3 - The Melancholia

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