THE CULTURAL IMPACT OF MATHEMATICS

UNIT II : MATHEMATICS AND MUSIC

Chapter 3 - Analysis of a Masterwork

In the previous chapter, a simple song was pulled apart to reveal the essential variables that enter into the composition of music in western civilization. Each of these variables was built upon mathematical foundations that were reducible to patterns of motion dependent on time relationships. At the micro-aural level there is the structure of tone explained through an understanding of periodic motions. On the macro-aural level we have the perceptible proportions of rhythmical movement. The objective of this chapter is to see how the variables function by extension to create patterns of musical form on a large scale. To do so, the first movement of Tchaikovsky's Symphony No.6 in B minor, known as his "Pathetique" symphony (Figure 1) has been selected for extensive analysis. It must be heard in its entirety before proceeding. (S14)
(L1): http://en.wikipedia.org/wiki/Tchaikovsky

The reader is encouraged to record their feelings while listening. Be particularly aware of changes that occur, dividing sections in the description as they naturally happen. Use musical or non-musical terms, whichever are appropriate in describing the effects. Attempt to create, as best you can, an architecture of the composition as it progresses. If necessary, listen to the movement more than once to confirm your impressions. You may wish to structure a diagram of some kind to order your thoughts. When this exercise is completed, continue with the discussion below.

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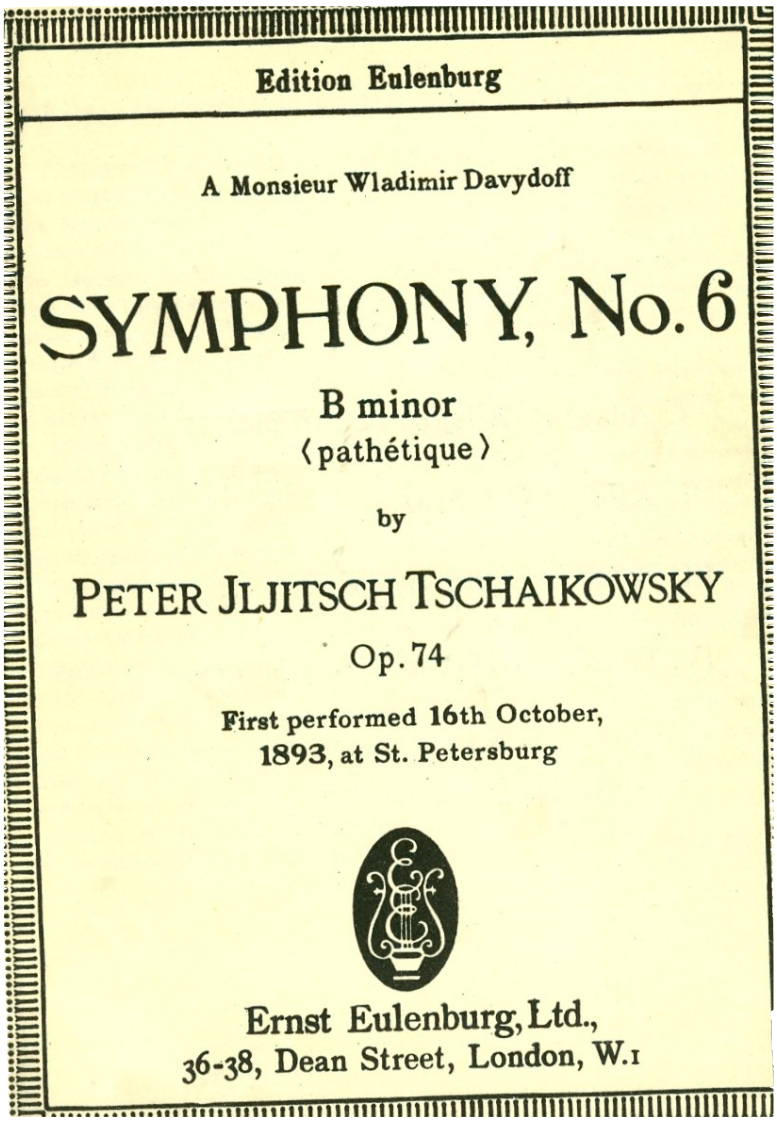
Figure 1


Edition Eulenburg
A Monsieur Wladimir Davydoff
SYMPHONY, No. 6
B minor
⟨pathétique⟩
by
PETER JLJITSCH TSCHAIKOWSKY
Op. 74
First performed 16th October,
1893, at St. Petersburg
Ernst Eulenburg, Ltd.,
36-38, Dean Street, London, W.1

When a comparison of listener reactions to the first movement of the Tchaikovsky symphony is made, agreement is generally reached about the following. The composition is dramatic, changeable, and intensely emotional. The movement opens with a slow, somber and mysterious introduction. This is followed by a faster section of considerable turbulence that builds up to a climax and then subsides to make way for the presentation of a slow and moving melody. A short dance-like passage intervenes which is followed by another presentation of the melody. The music dies away on this theme to a complete silence. This silence is shattered by a crashing chord and a long period of nervous movement, the music being pushed and pulled in all directions. The intensity gathers momentum until it reaches a broad and expansive thundering intonation in the brass instruments that signals the high point of dramatic expression in the work. When this concludes in a resolution of complete despair, the lyrical melody is heard once more even more intense and poignant after what had preceded it. Finally, the movement concluded in a passage which conveyed a sense of peace.

The main features we remember coincide with the divisions in the formal structure of the music. We may liken these events to taking a long trip by automobile and recording in detail some of the landscape passed along the way. A musical composition does resemble such a trip, but no distance is traveled in the sense of moving a physical object from one place to another. There is time involved; in this case about 15 minutes. But, just as in living, the passage of time is relative to the activity that fills it.

Music presents a high degree of information within a limited span of time, just as a visual work of art compresses a world of meaning into a limited field of space. The key to understanding music as an ordering of time is its motion. This is motion that is heard. We can read a score and imagine

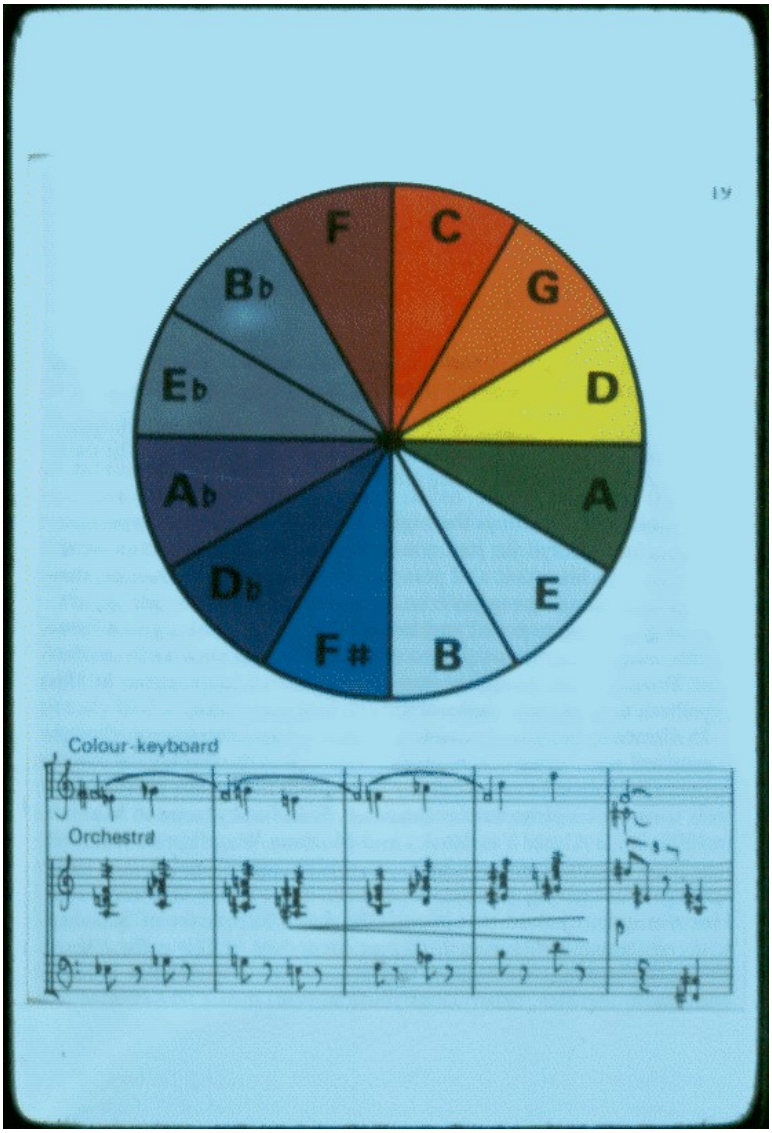
that we are "seeing" music in motion, but this is the same as reading an account of an automobile trip without having been in the car to experience it. Some people use their imaginations in listening to music and actually dream up visual images moving in coordination with the aural sense impressions they receive. For example, the composer, Scriabin,
(L2) : http://en.wikipedia.org/wiki/Scriabin#Influence_of_colour
reported that he actually saw a kaleidoscope of colors when he listened to music. He even devoted himself to the development of a color organ that called for colors to be connected with individual tones that could be projected on to a screen while a composition was played. (Figure 2)

This concept was imaginatively brought to the screen by the artists of Walt Disney studios in the 1940 film, Fantasia. The opening musical sequence presents an abstract rendering in colors and shapes of Johann Sebastian Bach's Toccata and Fugue in d minor as orchestrated by the conductor of the Philadelphia Orchestra, Leopold Stokowski. (L3) Recently, the film was restored and updated in stereophonic sound and still remains one of the most important innovations in movie film history. (L4) For many individuals, including the present author, it remains their first introduction to the world of classical music.

L3: http://en.wikipedia.org/wiki/Stokowski

L4: http://en.wikipedia.org/wiki/Fantasia_(film

Figure 2



The idea that music is motion is confirmed by the dancer (Figure 3) who translates it into visual patterns involving the human body. Dancing is undoubtedly the most direct response we can give to the musical stimulus

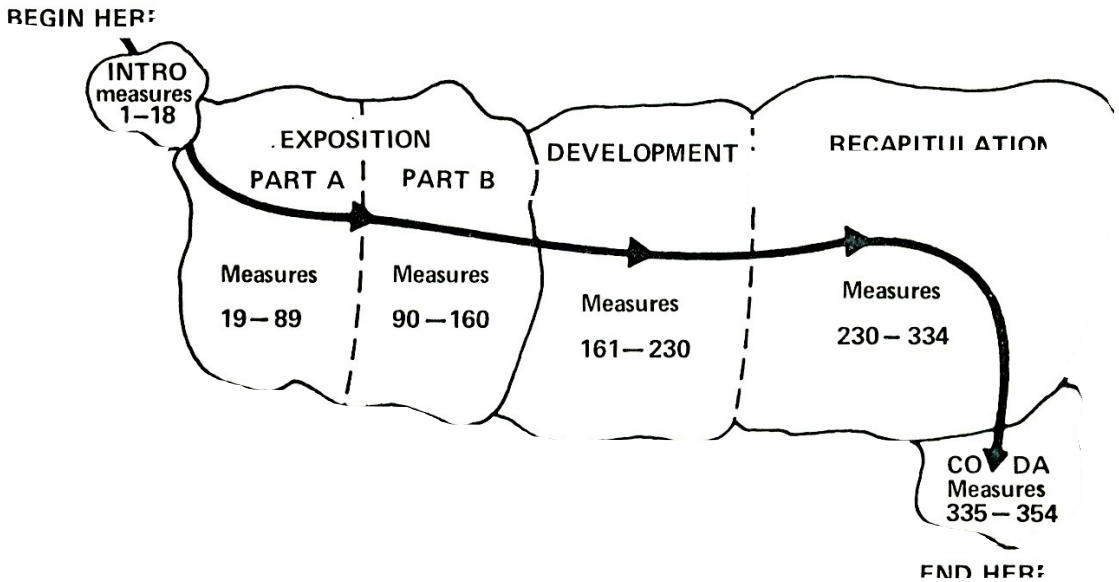
Figure 3

and it is no accident that dance music is the most popular in every culture. Although Tchaikovsky wrote several ballets, the "Pathetique" symphony is categorized as "absolute" music, that is, music which has its own intrinsic meaning and is not dependent upon literary or other non-musical references for its understanding. This music is not simple enough for social dancing and its complexity would challenge the best choreographers. Despite its intricacy, this is music which has a direct emotional appeal and even without musical training its structure is intelligible.

To aid the reader in this analysis, a "map" of the composition has been drawn outlining the main areas traversed (Figure 4). We may envision the musical form as a series of connected territories.

Figure 4

Figure 4: Formal Structure of Tchaikowsky's Sixth Symphony: First Movemen



Sonata Form (L5)

The first movement of the symphony is in "sonata form," or as it is sometimes called, "sonata-allegro form." This form was introduced in the 18th century and reached standardization particularly in the symphonies of Haydn and Mozart. Traditionally, the first movements of symphonies and solo pieces called "sonatas" are cast in this form. Students of the drama will see a striking similarity between this structure and what is known as "dramatic form." As in a play, dramatic form begins by introducing the characters and plot essentials. These are developed in the following acts to reach a climax and final resolution to the conflicts that appeared as foundations of the drama.

Tchaikovsky as much as any composer of the 19th century knew how to exploit sonata form for dramatic purposes. One of his famous compositions, the Romeo and Juliet Fantasy-Overture, shows a direct link between the action of Shakespeare's play and the sonata form. We have evidence in his personal correspondence also that he had a dramatic program in mind when he conceived the Sixth Symphony. He never revealed this program, preferring to let the music stand on its merits alone. With the overall picture of the sonata form in mind we shall begin to analyze the details of each section to see how continuity of motion is achieved and, in particular, how the separate sections fit together logically.

L5: http://en.wikipedia.org/wiki/Sonata_form

Motives (L6)

The short introduction to the movement is important for introducing two fragments of melodic material known as "motives." From these motives all of the essential melodies of the movement are constructed. The first motive is heard in the opening solo of the bassoon; a four-note figure graphed in (Figure 5). It rises on three successive tones of the scale, and ends by descending one step. The motive is repeated four times, each successive repetition beginning on the last tone of the previous statement. This has the effect of transposing the whole motive one scale step higher at each repetition. At the end of the fourth repetition, the phrase is ended with a cadence, the type of harmonic resolution discussed in Chapter 2. (S15) Figure 5

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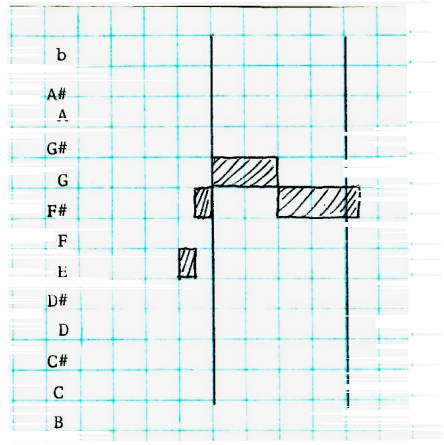


Figure 5: Tchaikowsky's Sixth Symphony, First Movement: Motive

The second motive is heard at the end of the Introduction (Measures 15-18). It is a simple descending scale intoned by the violas with a slight melodic turn at the end of its phrase. It contains a rhythm within itself that has important implications. This is a "dotted rhythm," similar to the opening of the "Do-Re-Mi" song. It is underlined for emphasis in the graph of (Figure 6). (S16)http://www.culturalmath.com/media/Sound-16.mp3

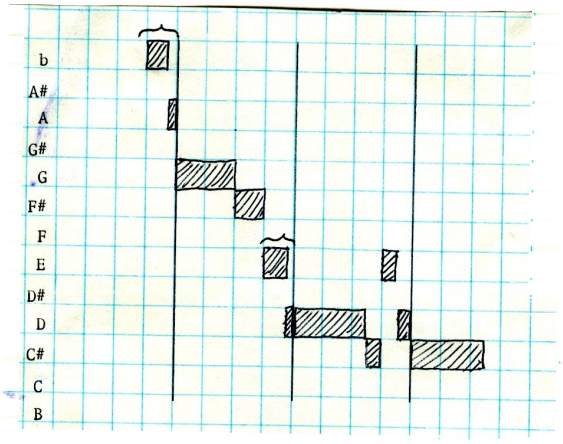
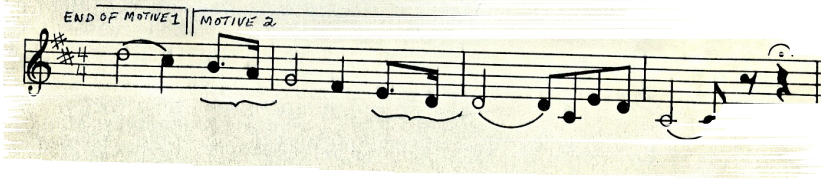


Figure 6 : Tchaikowsky's Sixth Symphony, First Movement: Motive 1



We have in these two motives, the kernel of the entire movement. They will be extended, varied, and combined. Their contrast and interplay will form the warp and woof of the musical fabric. A rising line and a descending line; nothing could be more general in melodic structure. There is a close parallel suggested here to mathematical systems. A good argument could be made showing that motives function in music in the same way that axioms do in mathematics. For, whatever follows in this music can be shown to be a logical consequence of the motivic structure.

Rising melodies lines are identified with increasing tension, the input of energy into the system. Conversely, descending in pitch releases energy and decreases tension. Now, consider this in relation to emotional states and the reader will have some inkling of why music has been called "the language of the emotions." It's no wonder also that the words motion and emotion are so close in structure.

L6: http://en.wikipedia.org/wiki/Motif_(music

The Key of B Minor

The Introduction section establishes another important setting for the movement; it defines the harmonic background. Referring again to the sequence of fifths:

••• Fb,Cb,Gb,Db,Ab,Eb,Bb,F,C,G,D,A,E,B,F#,C#,G#,D#,A#,E#,B#,~ ••

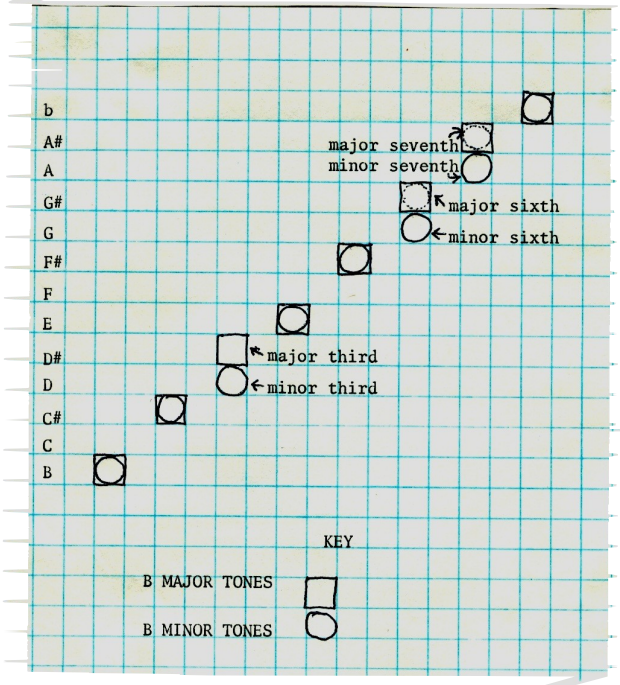
We see that in contrast to the C tonal center for "Do-Re-Mi", the Tonic for the symphony is B. This is located at a much higher level of harmonic tension than C. The Dominant and Subdominant relationships are also shifted upward; the Dominant of B is F# and the Subdominant of B is E,

the two tones directly adjacent to B in the sequence of fifths. In addition to this higher level of tension determined by the sequence, the first movement of the symphony is in B minor. The minor keys are associated in our minds with the darker emotions; that of brooding, violence, agitation, and melancholy. This is in marked contrast to the major keys which have a bright open quality that is most suitable for lighter and happier themes. No one has been successful in explaining satisfactorily the reasons why there is such a different psychological effect between major and minor. The two scales of B minor and B major are shown together graphically (Figure 7).

To appreciate the difference between B Major and B Minor listen to the last two preludes from Bach's Well Tempered Clavichord Book 1. This work traverses, in separate preludes and fugues, all of the major and minor keys from C to B, a total of 24 compositions. The first prelude in B Major (S17) contains 5 sharps, coinciding with all of the black keys on the piano, as well as the notes B and E (refer to Figure 7). As it moves gracefully through its lovely thematic material, it touches on minor chords, but is generally relaxed and joyful.
http://www.culturalmath.com/media/Sound-17.mp3

The second prelude in B Minor (S18) strides forcefully with an under lying scale like pattern in the bass. There is a more serious intent to this prelude, as it strives to break out of its minor mode. Indeed, there are distinct efforts to make use of the melodic minor with its leading sharp notes. In spite of the tempos of both preludes being close, the minor prelude is triple the length of the major. It is interesting to note that in all of the preludes and fugues in Book 1, Bach ends them with the Major Triad, the "chord of nature." http://www.culturalmath.com/media/Sound-18.mp3

Figure



Note: Dotted circles indicate tones that appear in the alternate versions of the minor key. These versions are known as the "harmonic minor" and the "melodic minor".

The principal differences are the appearances of the tones D#, the Major Third in the Major key and D, the minor third in the minor key. Also, the minor key frequently contains the tones G, the minor sixth, and A, the minor seventh, as opposed to G# and A# which always appear in the B major scale. The critical difference, however, is the opposition of the major and minor thirds.

One of the theories that is generally favored derives from the structure of the overtones discussed in Chapter 1. When the tone B is produced, the first six harmonics (overtones) include the following tone representatives: B, B,F#, B, D#, F#. The last three tones B, D#, F# constitute what is called the Major Triad built upon the root B. Because of its natural occurrence, it is sometimes referred to as the "chord of nature." It is also the fundamental triad for defining the Tonic B. The natural overtones therefore acoustically reinforce the stability of the B major scale. On the other hand, the B minor triad contains the tone D which is a semitone lower than the naturally occurring overtone D#. The theory asserts that the tension of the minor key is caused by the contradiction between the tone D that we hear produced in the minor scale and the tone D# that nature wants us to hear by virtue of the overtone series. The present writer acknowledges the acoustical importance of the overtone series, but feels that the psychological differences between major and minor are more a direct result of the stylistic treatment of the melodic and harmonic tendencies of the two scales.

For example, it is more usual in tonal music for D# to move melodically to the tone E while it is more common for D to move melodically to the tone C#. This melodic movement is also reinforced by a difference in harmony. The D# to E usually implies a Tonic to Subdominant motion, while the D to C# is more often a Tonic to Dominant motion. These are matters,

of course, for the advanced student in musical composition. Our main concern here is to note the distinction between major and minor in general.

The dark and somber Introduction to the symphony is harmonically a movement from the Subdominant root E to the Dominant root F#. The two principal motives then are introduced over an extended cadence lasting some l8 measures. We can now understand why we have the feeling of expectancy. Harmonically the Introduction prepares the way for the Exposition section to begin the basic key, B minor. This kind of connection between the Introduction and Exposition is a fundamental characteristic of classical sonata form. The great majority of symphonies from Haydn to Tchaikovsky begin with a slow introduction that raises the listener's expectations of the exposition to come. This is a composer's way of heightening the drama.

Tchaikovsky doesn't disappoint us; the Exposition begins with a doubling of tempo and the entry of the first theme over the Tonic root. Although the Introduction presented the basic motives, the Exposition constructs genuine melodic themes out of them. A close up of this section (Figure 8) shows that it is divided into two main segments. The first section is in a rapid tempo and it pits thematic material derived from both motives against each other. The second section begins in a much slower tempo and is devoted to the presentation of a famous lyrical theme. Analysis will show that this theme is principally constructed out of Motive 2, though each of its phrases is concluded with the first motive.

Figure 3

Tchaikowsky's Sixth Symphony. First Movement: Exposition Part I

Tempo: 116 beats per minute — moderately fas
Meter: 4 beats per measur
Length: 70 measure

Content:	Motive 1 measures 19 – 39	Transition measures 39 – 44	Motive 2 measures 44 – 50	Var. of Motive 2 measures 50 – 66	Motive 2 measures 67 – 88
Harmony:	Tonic — modulations begin in measure 30 with entry of violins	Relative Major key D major	Cycle of Fifths F# – C#	Cycle of Fifths C#-G#-D#	Modulation to Relative Major.
Highlights:	Buildup to climax in strings	Note woodwind entry on motive 2	Extensive development of new variant of motive 2		Entry of brass completes development and begins transition to Part II.

Tchaikowsky's Sixth Symphony. First Movement: Exposition Part I

Tempo: 69 beats per minute for melody — slow with motion
110 beats per minute for Bridge section — moderate
Meter: 4 beats per measur
Length: 72 measure

Content:	Lyrical theme measures 89 – 100	Bridge measures 101 – 129	Lyrical theme measures 130 – 160
Harmony:	Relative Major (D)	Cycle of Fifths, D - A - E	Relative Major (D)
Highlights:	The melody and its Romantic harmony	Canon imitation, Use of Motive 2, brass in rising line	Triplet accomp. Intensified doublings. Higher dynamic level, clarinet solo to end.
Song Form:	A	B	A

It is typical of sonata form to contain two strongly contrasted melodic ideas. The first section of the Exposition is dramatic and organic, charged with energy and growth. The contrasting lyrical theme of the second section is self-contained and intensely romantic. It rounds off the Exposition bringing it to a quiet close. This, in brief, is a skeletal picture of the Exposition; about 70 measures for the first part and 72 measures for the second.

Although the balance between the two sections is most obvious in their melodic and rhythmic differences, the principal contrast is attained through harmonic means. The process is one that was touched on briefly in the analysis of "Do-Re-Mi." It is known as modulation and it results in a recognizable change in the tonal center. If modulation is brief and transitory, it creates a feeling of instability, impelling the music to move forward to a more clearly defined harmonic goal. This is what occurs in the rapidly moving first section of the Exposition. (S19)

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The key of B minor is quickly abandoned as the composer organically develops the first theme. The primary goal is the key of D major, the harmonic foundation for the lyrical theme. Indeed, the root D is briefly touched upon at the climax of the opening statement in measure 38, about halfway through the first section. (S20)

http://www.culturalmath.com/media/Sound-20.mp3

At this moment, Tchaikovsky even hints at the shape of the lyrical melody to come with a brief appearance of the downward moving scale of Motive 2 in the woodwinds. However, the harmony rapidly changes under this scale as the motives vie with each other contrapuntally.

The composer builds the harmonic tension through rapid changes of tonic roots built upon the sequence of fifths. The movement is from F# to C# to G# to D#. At the climactic point in this sequence the texture is abruptly changed with the first entry of the brass instruments which strongly state Motive 1 against the agitated figuration in the woodwinds and strings. Instead of continuing upward in the sequence of fifths to A#, there is a jolting change at this moment in the direction of the root A. Three measures later this is reinforced by the first rolling of the kettledrums. It is the tone A. This is the signal for the transition to begin for movement to the second section, for A is the Dominant of the Tonic goal D. (S21)

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It takes 20 measures to reduce the momentum and clearly establish the key of D major. Throughout the first section, minor sounds have predominated, unrelenting in their tension. Now, in a sort of mirror reflection of what occurred at the end of the Introduction, the violas once again preface the next section of the composition. They do so with an inverted form of the second motive, sung at the same slow tempo as in the Introduction, and, as we have seen, transposition allows a choice of any octave for setting the melodic line. The violins and cellos are muted to present a quiet presentation of the theme. (S22)

http://www.culturalmath.com/media/Sound-22.mp3

A single melodic line is two dimensional in its expression. When accompanied by its harmonic content, a third dimension is added to its meaning. This is analogous to the distinction between graphs in the xy plane and those that are portrayed in three dimensions with a third axis (Figures 9 & 10).

Figure 4

Function of a single variable

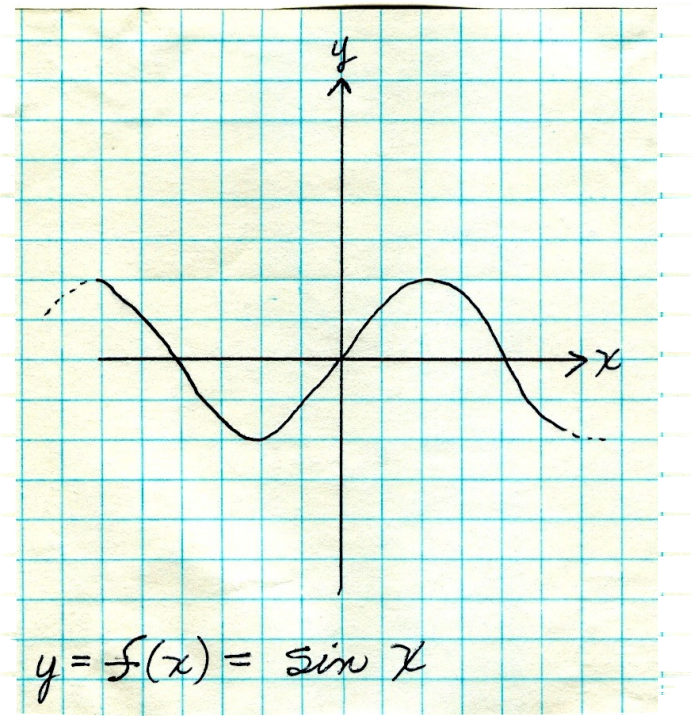
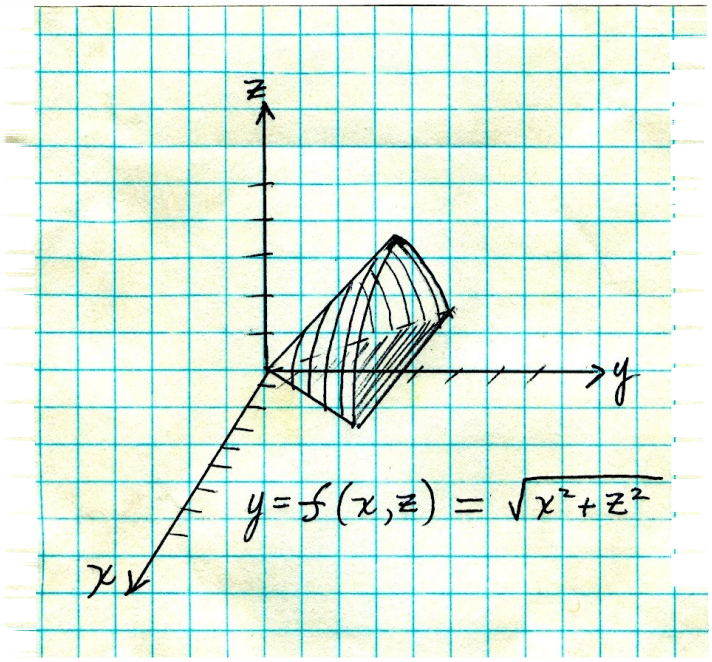


Figure 1(

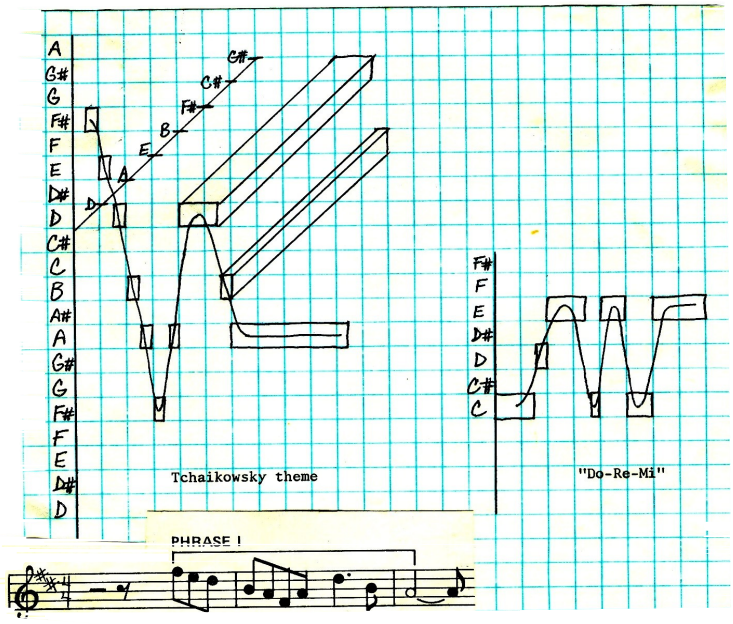
Function of two variables



By way of contrast, the first phrase of "Do-Re-Mi" and the opening phrase of the Tchaikovsky melody have been set along side each other in (Figure 11). The graphs reveal some interesting distinctions. Where "Do-Re-Mi" moves upward in pitch, the Tchaikovsky theme has a general downward curve. The range of pitches for "Do-Re-Mi" is small in comparison to the symphonic theme, the latter encompassing an octave. The Tonic root underpins both phrases. The Tchaikovsky theme, on the other hand, contains harmonically intense chords which contrast sharply with the Tonic root as shown graphically by the third dimension extensions. These chords have the effect of introducing ambiguities into the direction of harmonic movement. The result is that the symphonic theme tends to alternate between major and minor sounds, a technique that increases the poignancy of the theme.

Figure 11

Contrasts in the graphs of the opening phrases of the "Do-Re-Mi" song and the symphonic theme by Tchaikowsky



The second halves of both melodies are even more startling in their contrasts (Figure 12). Where the song begins its second half one scale step lower than the end of its first half, the symphonic theme leaps a whole octave for its second half. The result is an intensification that is harmonically reinforced by a dissonant chord, distantly related to the Tonic. In addition the second halves of both melodies continue the opposed motions noted in their openings. This contributes considerably to the overall differences in their psychological effects. (S23)

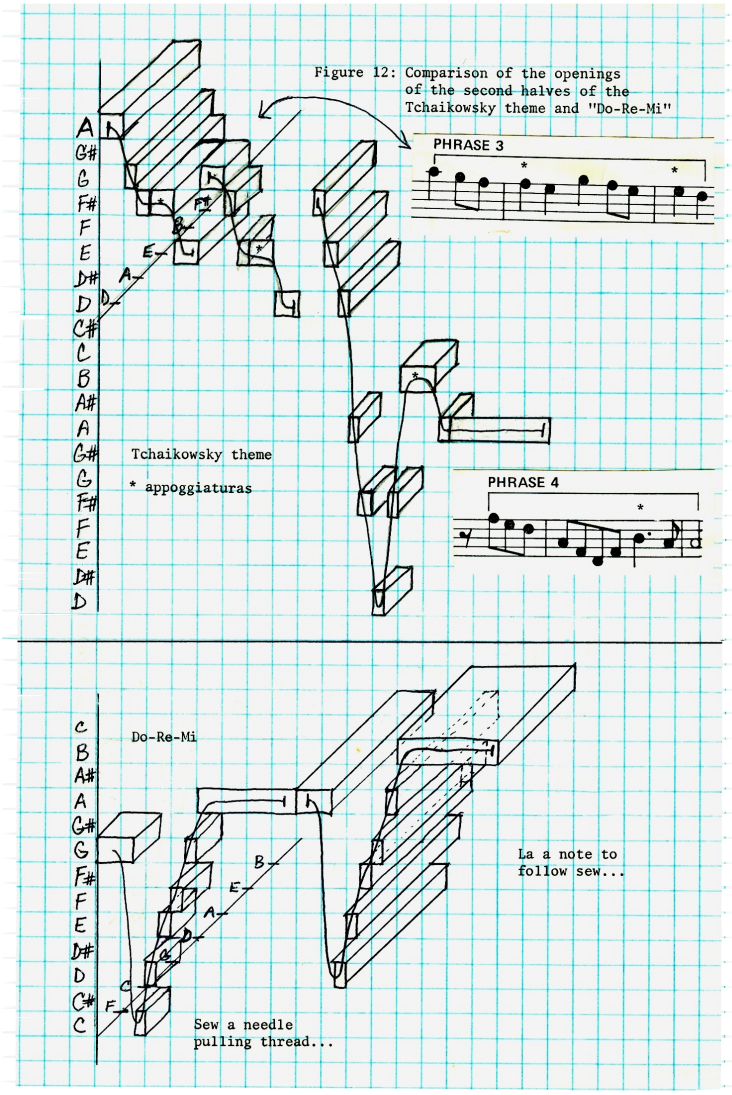


Figure 12: Comparison of the openings of the second halves of the Tchaikowsky theme and "Do-Re-Mi"

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Relative Keys

Before moving on in the analysis of the symphony, it is important to notice how closely the keys of B minor and D major are related. They both have the same basic complement of scale tones. This makes modulation from one to the other very simple. Because of this D major is called the "relative" major of B minor and vice versa. We noted that there was a certain ambiguity in the lyrical theme. Part of this is due to the appearance of dissonant chords which could just as easily resolve into B minor as to D major. This is coupled with the prominent appearance of the tone B in the melody, especially as a rhythmically and harmonically stressed sound, known as an "appogiatura." It is a very common occurrence in sonata movements which are harmonically built upon a minor key to have their lyrically contrasting themes in the relative major key.

The Bridge (Measures 101-129)

Having presented a melody as beautiful as this one, there is a natural desire to repeat it. However, it would be poor artistic judgment to do so immediately. Instead, the composer increases the desire for repetition by creating an interlude known as a "bridge." For a composer of Tchaikovsky's caliber, the construction of a bridge can be as interesting as the melody it leads to. The elements composing the bridge are in complete contrast to the melody. In fact, the bridge has several of the features analogous to the exhilaration of the "Do-Re-Mi" song. The tempo is picked up with a buoyant rhythmic accompaniment in the strings. Motive 1 is extended into a rising

scale of triplets (groups of three tones heard for each beat) rounded off by a short descent. This melodic line is played as a canon in the woodwinds.

This simple imitative technique is one of the oldest devices to be heard in western music. It is familiar to us in the singing of "Row, Row, Row Your Boat" as a round. After the canon is repeated twice, the bridge theme is completed with a phrase echoing the last half of the lyrical melody, the short dotted rhythm we called attention to in the analysis of Motive 2. This downward moving phrase is magnificently counterpointed with a rising scale line in the brass instruments (Measure 105-108). (S24)

http://www.culturalmath.com/media/Sound-24.mp3

There are three repetitions of this bridge theme, each repetition varied by harmonic progressions through the sequence of fifths; from D to A to E, again a rising line of harmonic tension that is identical to the sequence in the second half of the "Do-Re-Mi" song. We can see how powerful a generalization this kind of harmonic movement is in tonality, through its application in such disparate compositions.

At the end of the bridge, Tchaikovsky makes use of the same kind of modulation as Rodgers to "bring us back to Do." The tempo is slowed again and the lyrical theme returns.

Before we leave this bridge, let's glance back at it for a moment, if only to pay our respects. compositions of the length we are analyzing will inevitably have connecting passages like this one. Yet, these are the places where a composer's true genius is tested. These 29 measures fit themselves into the symphony with perfect integration and make this section of the Exposition into a fine example of what is called “song form." Symbolically, this is described as A B A from, the lyrical melody being the A part and

the bridge corresponding to the B part. So here we have an example of a small form, the song, incorporated within a large form, the sonata.

In the repetition of the melody, we hear again the composer's refusal to repeat himself verbatim. The strings remove their mutes and the full orchestra enters with a throbbing undercurrent of triplets. The violins and violas play the melody in unison across a three-octave span. This big spread intensifies the melody and the underlying harmonies reveal its passionate expression. (S25) http://www.culturalmath.com/media/Sound-25.mp3

The frequent use of dissonant chromatic harmonies was typical of the Romantic period and still finds common usage as movie background music. In the next chapter we shall find that it was precisely this treatment of harmony that other 20th century composers were to reject in their search for fresh means of expression. Many of them were to return to the simple clarity of the earlier modal styles to create an astringent, unemotional style known as neo-classicism.

To return to the music, after the restatement of the lyrical melody, there is a long extension of the melodic elements in a descending line which gradually subdues the intensity and leads to the quiet enunciation of the theme in the clarinet. The Exposition ends on a slow descending D major chord, and all is peace. (S26)

http://www.culturalmath.com/media/Sound-26.mp3

The Development Section (Measures 161-230) (Figure 13)

The peace is shattered by the loud dissonant chord which begins the development. This is one of the more startling changes of mood in music of the Romantic era. Loudness, of course, creates its own tension by virtue of the increased energy that is put into the system. But the opening chord is

also described as dissonant, and this factor increases the tension level beyond that imposed by dynamics. (S27)
http://www.culturalmath.com/media/Sound-27.mp3

Figure 1:

Tchaikowsky's Sixth Symphony, First Movement: Development

Tempo: 144 beats per minute — very fast, basic unit is eighth note
Meter: 4 beats (quarter notes) per measure
Length: 69 measures

Content:	Introduction measures 161 - 170	Motive 1 measures 170 - 189	Motive 2 measures 190 - 197	Transition measures 198 - 229
Harmony:	Modulatory minor tonics	Cycle of Fifths d-a-e-b	Transition back to d minor	Chromatic melodic movement, unstable harmonies
Highlights:	Loud dissonant chords	Agitated motion	Loud brass statement of Motive 2	Syncopated line in strings

Consonance and Dissonance

The understanding of consonance and dissonance has never been satisfactorily resolved. Pythagoras and many of the theorists who followed him believed that the frequency ratios between tones was the explanation for their consonant or dissonant relationship. This theory says that the harmonic interval of the octave is most consonant because the ratio between the tone frequencies is the simplest, 1:2. The idea behind this mathematical theory is that the smaller the numbers constituting the ratio, the more consonant the interval. If we arrange the intervals according to this criterion (Figure 14), the next most consonant interval after the octave is the perfect fifth, 2:3; then the fourth, 3:4, and so forth until we pass to the higher ratios and a feeling of dissonance. In general we do seem to hear more tension in the intervals as we proceed to higher numbers relating the constituent tones. The main objection to this theory lies in the fact that in actual musical practice, the intervals beyond the octave are seldom produced in the precise mathematical ratios given. Yet, despite deviations from these ratios, we are still able to recognize these intervals and respond to their relative levels of tension. More recently better explanations of consonance and dissonance have been put forward. The most convincing have been the theory of "beats" proposed by Helmholtz, (L7) and Stumpf's fusion theory (L8) based upon psychological perception. We will not attempt to explain these theories here. The reader who is curious should investigate the literature on the subject. Rather, we shall approach the problem from a pragmatic point of view.

L7: http://en.wikipedia.org/wiki/Helmholtz#Acoustics_and_aesthetics

L8: http://en.wikipedia.org/wiki/Psychoacoustics

Figure 14

Harmonic Intervals and their Defined Ratios*

Interval	Frequency Ratios
Unison	1 : 1
Octave	2 : 1
Perfect Fifth	3 : 2
Perfect Fourth	4 : 3
Major Third	5 : 4
Major Sixth	5 : 3
minor third	6 : 5
minor sixth	8 : 5
Major Second	9 : 8 or 10 : 9
minor seventh	9 : 5 or 16 : 9
minor second	16 : 15
Major Seventh	15 : 8
Tritone (diminished fifth)	25 : 18

* Ratios determined from the positions of the constituent tones in the overtone series

The dissonance of the chord which opens the Development section derives its dissonant content from the context of the tonal relationships within which the composer restricts himself.

The opening chord of the Development is built upon A, the Dominant of the Tonic root D which had ended the Exposition. The constituent tones of the chord are A,C, Eb, and G. The sound is intrinsically minor. The minor thirds A to C, and C to Eb are responsible for this effect. The dissonant bite of the chord is created by the diminished fifth between A and Eb, and the interval of the major second between the G and A (refer to the table of intervals in Figure 14).

Tchaikovsky was a master of orchestration. He achieves the cutting edge of the chord by scoring the seconds in the oboes, clarinets, horns,

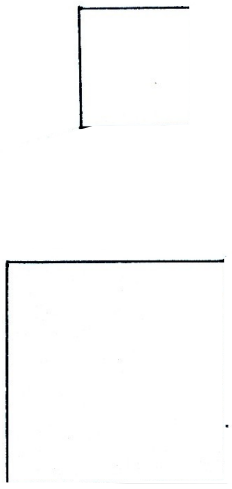
and trombones. The highest tone we hear is the Eb, scored in the flutes, trumpets, and violins. Thus, tones which contribute most to the dissonant sound are heard most prominently.

Transformations

The opening dissonant chords are a prelude to the reentry of Motive 1. When it appears (Measure 170) the Development section properly begins. The main idea underlying a Development is that of transformation. The variations encountered in the shaping of the motives earlier are now increased at an accelerated rate. Transformation in mathematics is a fundamental idea. It is encountered for example in the process of similarity. In (Figure 15), the smaller square is transformed into a larger square four times its area by a doubling of its sides. Likewise, on a more complex level, perspective in the visual arts is achieved by a set of transformations governing projection and section. It could be argued with good reason that music is the art of transforming patterns of motion.

Figure 15

Figure 1:

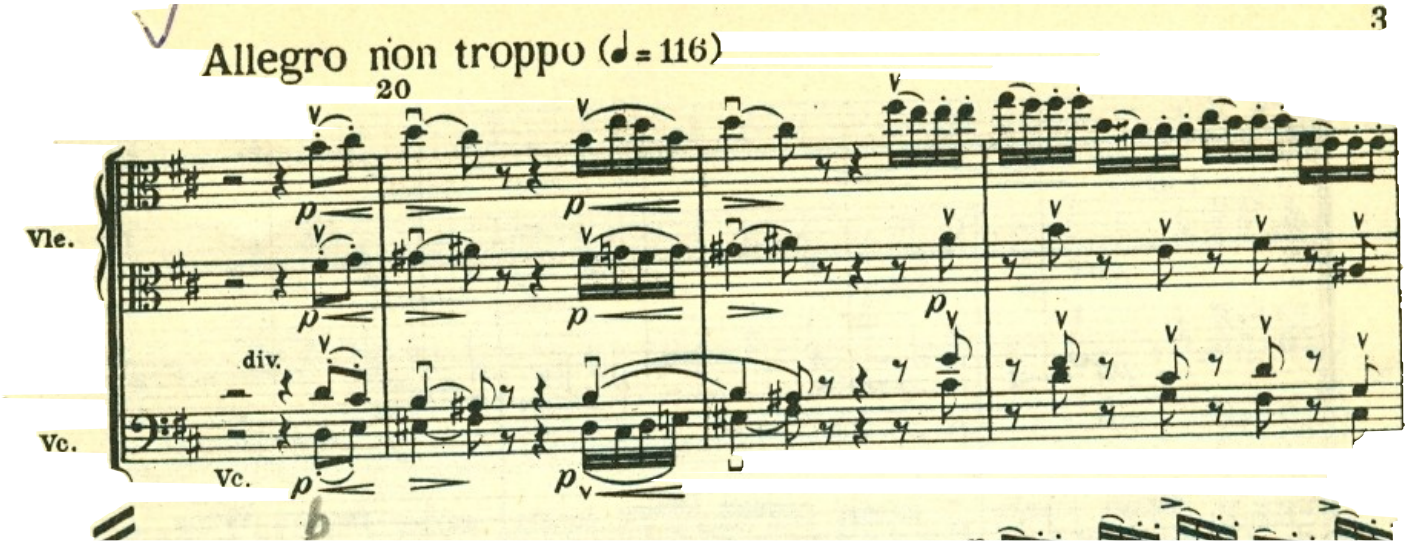


Transformation of a smaller square into a larger square four times its area by a doubling of its sides

As an example, consider the opening theme of the Exposition: (S28)

http://www.culturalmath.com/media/Sound-28.mp3

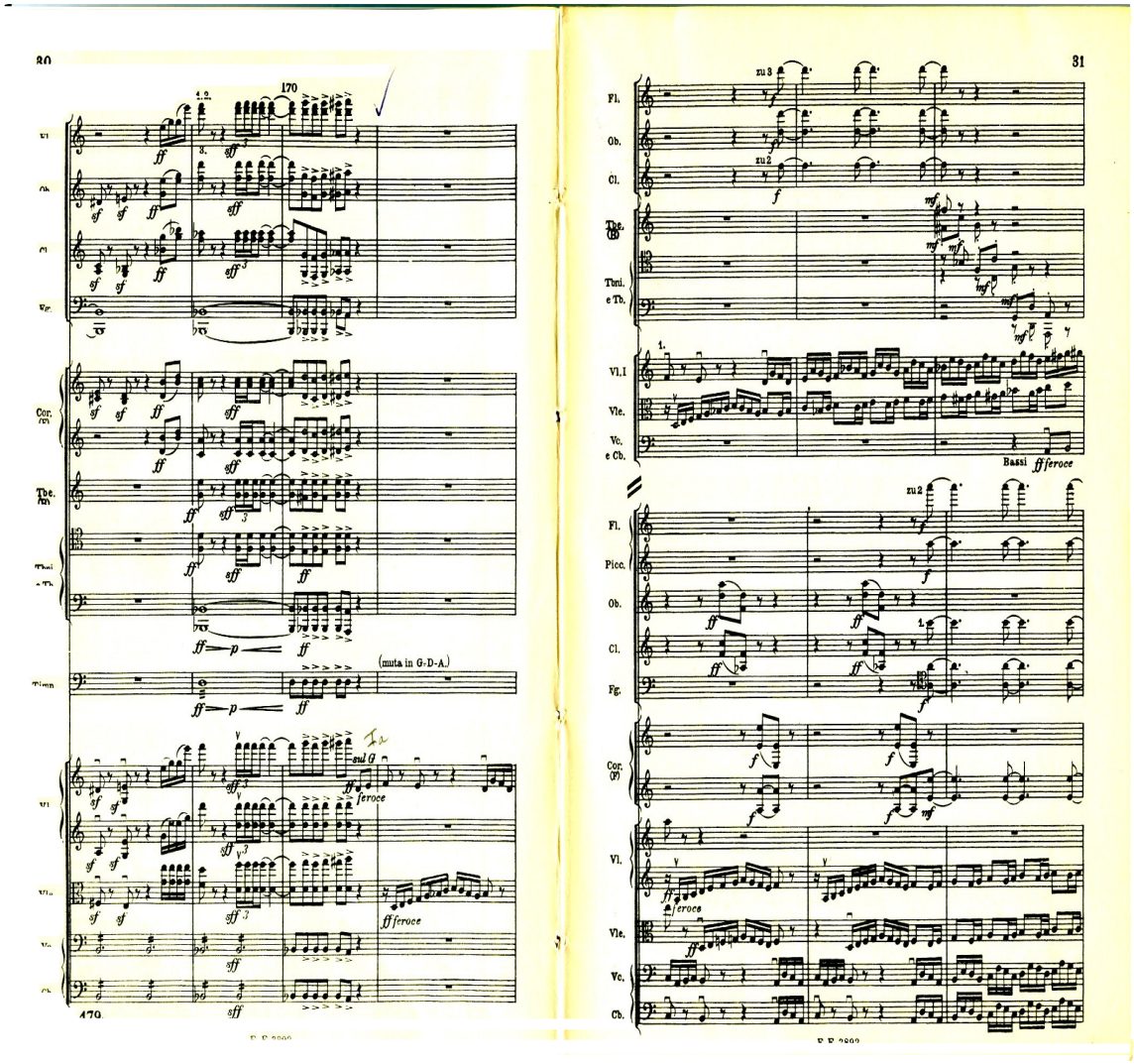
Figure 16



The Development begins in the same way, but immediately builds on the rapidly turning figure in a rising line of rhythmic ferocity. (S29)

http://www.culturalmath.com/media/Sound-29.mp3

Figure 17



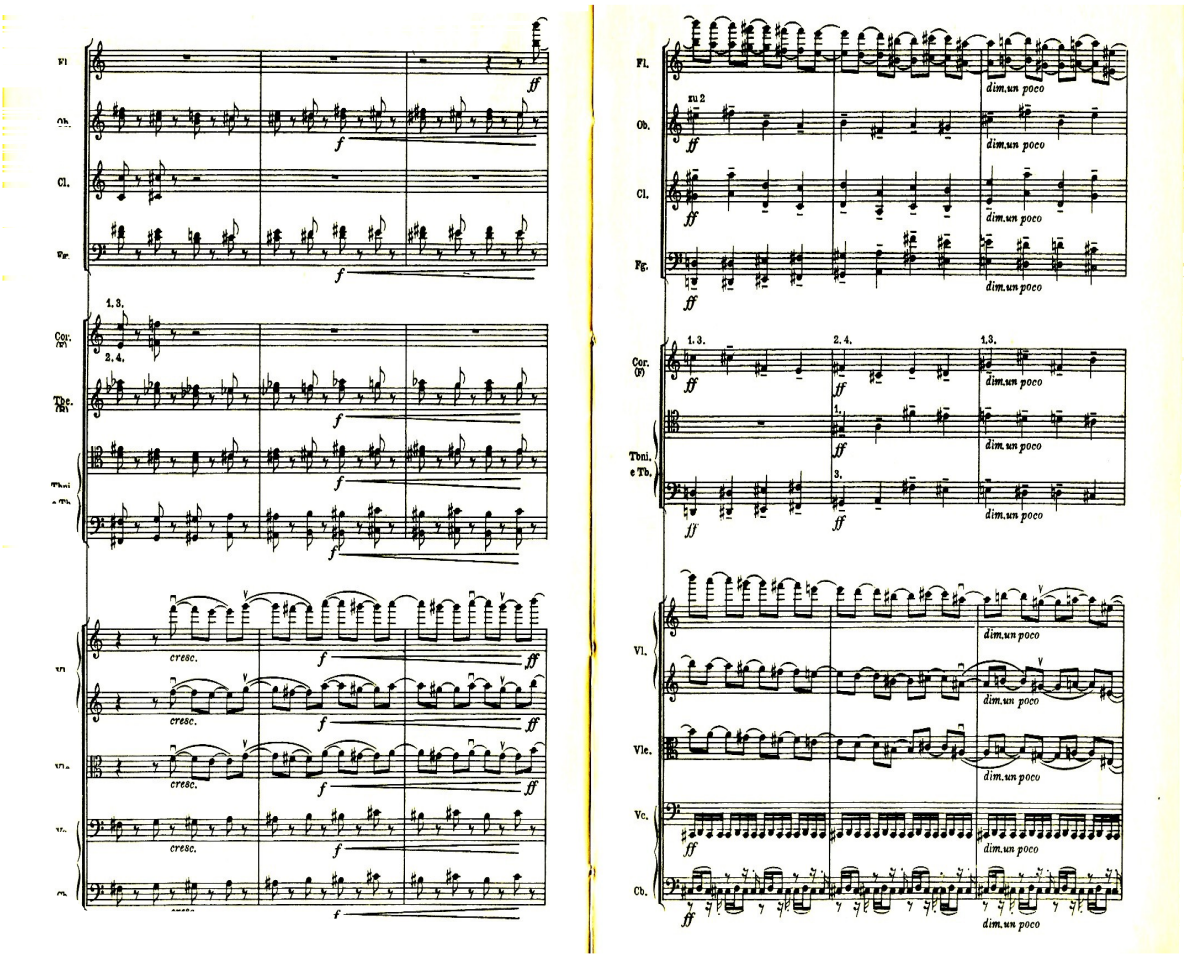
Once again this construction is based upon the sequence of fifths, another transformation with a mathematical foundation. In contrast to the end of the Exposition in D major, this sequence begins in D minor. It moves rapidly through A to E to B minor, bringing us back to the main tonal center of the composition. At this point, a rising line of imitation occurs between the strings and woodwinds building to the entry of the trumpets (Measure 190). The harmonic background is again returned to D minor as the trumpets intone the second part of the lyrical melody which began with the octave leap. This phrase is even more dramatically incisive being cast in the minor scale and played as counterpoint against the Motive 1 figuration. It, too, is transformed into a long descending scale line that ends abruptly in two short eighth notes (Measure 197). (S30)

http://www.culturalmath.com/media/Sound-30.mp3

At this point, the turning figure is transferred to the lower instruments and transformed into a triplet accompaniment figure in preparation for another melodic development (Measure 201). Looking again at Motive 1, the appoggiatura effect is now transformed into a melodic sequence that Tchaikovsky frequently used. It is described as an anticipation and is related to the rhythmic effect of syncopation, the shifting of accent to a weak beat. This is first heard in Measure 207 and dominates the remaining part of the Development. By Measure 230 the French horns are left with this syncopation in triplets. It provides a quiet agitated movement underlying the beginning of the Recapitulation. (Figure 18) (S31)

http://www.culturalmath.com/media/Sound-31.mp3

Figure 18



The Dramatic Climax

At this point the music commences what is probably the most extraordinary part of the entire movement. One hundred years before the composition of this work the sonata form had been fairly well standardized in the symphonies of Mozart (L9). We call that time the classical period in music because a perfect balance and proportion was achieved in the musical forms that were used. This perfection can be traced back to Ancient Greece whose ideals had a profound impact on artistic creativity during Mozart's time. Beginning with Beethoven (L10) in the early 19th century, these ideals were evolved into the Romantic spirit which looked upon form as being subordinate to content. Tchaikovsky idolized Mozart, but he was a true believer in the Romantic expression of his own time. It is in this section of the composition that Tchaikovsky's Romantic leanings reign supreme. (Figure 19).

L9: http://en.wikipedia.org/wiki/Mozart

L10: http://en.wikipedia.org/wiki/Beethoven

Figure 19

Figure 19

Tchaikowsky's Sixth Symphony, First Movement: Recapitulation

Tempo:	Same as in the development section			
Meter:	Same as in the development section			
Length:	75 measures			
Content:	Motive 1 measures 230 – 267	Motive 2 measures 267 – 276	Motive 2 measures 277 – 304	Lyrical theme measures 305 – 334
Harmony:	Unstable, continuation of chromatic development	Entry of Motive 2 coincides with arrival of dominant	Dominant	Tonic (B major)
Highlights:	Intensive development of Motive 1 in all parts and rhythmic contrasts	High point of furious crescendo released at this point.	Broad & intense climax of the movement.	Statement is intense and dramatic. Clarinet solo brings the section to a close.

Instead of following classical tradition which directed that the Recapitulation be a replica of the Exposition, Tchaikovsky chose to make the Recapitulation an extension of the Development. In this way, the music becomes truly organic, propelling growth of his ideas to reach for one of the most dramatic climaxes to be found in the symphonic repertoire.

The Recapitulation begins as expected with Motive 1 (Measure 230), but it has not returned to B minor as would be expected in classical sonata form. Instead, the motive is compressed into a chromatic half step rendition that underscores the lack of harmonic stability. All of this is symptomatic of an extension of the Development. This is to be the last and most furious presentation of the first theme. Motive 1 rises chromatically becoming more and more compressed and fragmented in a rapid crescendo. At Measure 244, the brass thunder in with Motive 1 and quickly alternate with the strings and woodwinds. Four measures later the woodwinds and strings are pitted against each other in fragments of the motive hurled like thunderbolts. Then the strings commence a long line of Motive 1 that reaches its climax at Measure 262 where the rhythmic propulsion of the triplets become predominant in the shaping of Motive 1 throughout the entire orchestra. This culminates at last in the entry of the brass with the dotted figure of Motive 2 over the Dominant F#. The descending scale leads to the beginning of a broad flowing crescendo based on Motive 2 starting at Measure 277. (S32)

http://www.culturalmath.com/media/Sound-32.mp3

Everything has been directed to this moment; it is one of the most anguished expressions of despair to be heard in symphonic music. We have reached the dramatic climax of the composition. At the height of this cres-

cendo (Measure 284), the strings and flutes reach for a high F# and playing as intensely as they can, slowly descend two and a half octaves on a B minor scale, carried forward by the trombones dramatically stating the phrase built upon the dotted rhythm of Motive 2.

The end of this section is like a descent into the tomb; the release of a tremendous weight; the cessation of motion. Harmonically we have reached the Dominant of B minor and dramatically we have come to the "catastrophe," in this instance the musical equivalent of death. (S33)

http://www.culturalmath.com/media/Sound-33.mp3

What can possibly follow death? The composer answers this deep philosophical question with a musical promise of resurrection and peace. Out of the depths of despair, the cellos and basses rise from F# to B, (Measure 305) and for the first time we hear the bright and hopeful key of B major as the lyrical melody returns. To enhance this quality of hope, the composer extends the rising line figure as a counterpoint against the main melody. When this subsides, the clarinet is heard again singing the melody as it did at the end of the Exposition. This time the solo brings the Recapitulation to a close. (S34)

http://www.culturalmath.com/media/Sound-34.mp3

We now hear the final section of the movement; a short but magnificent coda. Over a descending B major scale played pizzicato by the strings and repeated over and over again, the winds play a rising transformation of the dotted fragment of Motive 2 and the horns and trombones close the composition with a descending echo of the fragment. All is peace and repose as the journey is ended. (Figure 20). (S35)

http://www.culturalmath.com/media/Sound-35.mp3

Figure 20

Tchaikowsky's Sixth Symphony, First Movement: Coda

Tempo: 80 beats per minute — slow with motion
Meter: 4 beats per measure
Length: 20 measures

Content:	Motive 1 measures 335 - 344	Motive 2 measures 345 - 354
Harmony:	Subdominant (tonic scale ostinato)	Tonic closes in B major

Highlights: Peaceful close contrasts to forebodings of the Introduction section of the movement.

The last sentence is, of course, partially inaccurate. There are three more movements to this symphony, each of which must contribute to the total compositional idea. The second movement is in marked contrast; a lilting waltz-like theme, ingeniously written in Five-Four time. The third movement is one of the most rousing marches ever written, a tour de force in orchestral writing.

But, it is the last movement, a soul wrenching adagio, that truly completes the dramatic question posed in the opening movement. Its main theme is built out of the descending scale pattern of Motive 2 and it ends the symphony in the reflective sorrow of B minor. This was to be Tchaikovsky's epitaph, for a few days after the first performance of the symphony the composer died of cholera. A certain doubt exists as to whether his death was accidental or a suicide. Consciously or unconsciously, he drank a glass of water directly from the tap which was prohibited at that time because of the danger of contracting the disease.

Some people have read into the Pathetique Symphony a foreboding of this event. However, those who were with him at the time reported that he was in good spirits and actively planning new creative projects before he took the fateful glass of water. The Pathetique represented for Tchaikovsky a kind of rejuvenation. He was never more sure of his creative powers when he tackled this work and when he at last completed it. Even despite a disappointing first performance, he seemed to have no doubt about its inherent value.

In the analysis of any work of art care must be taken. In scientific investigation, "the whole is equal to the sum of its parts" is a basic axiom. In art the opposite is true. There are many works of art which are ingeniously planned and executed, but as a total conception they don't come off. In

these well wrought works that fail, there is something missing that goes beyond any system of analysis that we can devise. Call it soul, vision, or what you will, but, in any age, only a handful of artists are blessed with the insight that stamps their work as timeless and immortal. On the other hand, if they were not also endowed with a mastery of their craft, they would never be able to communicate the perfection of their inner vision. It is this unique combination of craftsmanship and profound insight that reaches total integration in the great artist. Unfortunately, this god-like perfection seldom occurs with consistency in the artistic output of any one individual.

The career of every first-rate artist is checkered with the production of second-rate creations. The ranking of artistic genius, then, becomes something of a statistical assessment where consistency of excellence is matched against the total body of works produced. Value judgments are particularly vulnerable to the outlook of the age in which they are made. For a good portion of the 20th century, for example, the music of Tchaikovsky was looked down upon by many so-called arbiters of good taste. One reason for this was the aesthetic reaction to Romanticism which took place around the turn of that century. Such reactions are a healthy stimulus to finding new directions in artistic expression. Johann Sebastian Bach's music fell out of favor and was almost forgotten for about one hundred years after his death. Yet, in a way this temporary eclipse of his music was necessary in order to have created the conditions for the evolution of the classical style of Haydn (L11) and Mozart.

L11: http://en.wikipedia.org/wiki/Haydn

The greatness of a work of art is not subject to the changing fads in artistic tastes. Tchaikovsky's Pathetique Symphony is, as he himself believed, his finest work. Within the dimensions of his tonal world, it is a perfectly conceived and executed composition. As long as we are able to project ourselves into his world, we will be moved by the power of its conception.

The foregoing analysis has illuminated some of the symphony's structure and logical coherence. It has also been attempted to help in understanding how craft and inspiration are molded together by a master. The reader is encouraged to put it back together again and hear it as the total experience that the composer intended. Only then will one be convinced of the artistic axiom that the whole is greater than its parts.

Masterwork Analysis Links

L1: http://en.wikipedia.org/wiki/Tchaikovsky

L2: http://en.wikipedia.org/wiki/Scriabin#Influence_of_colour

L3: http://en.wikipedia.org/wiki/Stokowski

L4: http://en.wikipedia.org/wiki/Fantasia_(film)

L5: http://en.wikipedia.org/wiki/Sonata_form

L6: http://en.wikipedia.org/wiki/Motif_(music)

L7: http://en.wikipedia.org/wiki/Helmholtz#Acoustics_and_aesthetics

L8: http://en.wikipedia.org/wiki/Psychoacoustics

L9: http://en.wikipedia.org/wiki/Mozart

L10: http://en.wikipedia.org/wiki/Beethoven

L11: http://en.wikipedia.org/wiki/Haydn

LIST OF RECORDINGS

S14 -S16:

TCHAIKOVSKY : SYMPHONY NO. 6 IN B MINOR OP. 74 “PATHETIQUE”
NEW YORK PHILHARMONIC, CONDUCTED BY DIMITRI MITROPOULOS, COLUMBIA ODYSSEY LP 32 16 0216

S17-S18:

BACH : WELL TEMPERED CLAVIER BK 1, PRELUDES IN B MAJOR AND B MINOR, GLENN GOULD, PIANIST, COLUMBIA LP ML 6176

S19-S35:

TCHAIKOVSKY : SYMPHONY NO. 6 IN B MINOR OP. 74 “PATHETIQUE”
NEW YORK PHILHARMONIC, CONDUCTED BY DIMITRI MITROPOULOS, COLUMBIA ODYSSEY LP 32 16 0216