THE CULTURAL IMPACT OF MATHEMATICS

UNIT II

MATHEMATICS AND MUSIC

Chapter 4 - Contemporary Musical Structures

The twentieth century witnessed such a violent upheaval in the conditions of human existence that, even now, we find historians still trying to assimilate the data into some coherent framework. This is particularly true in the arts which respond directly to the stimuli of their cultural environment. Tchaikovsky's last symphony was composed in the twilight of the Romantic period. A little more than twenty years after his death, the world he knew would be crushed into oblivion on the battlefields of World War I. Even as the culture he lived in contained the seeds of its own destruction, so too did the music of his generation give birth to stylistic tendencies that would eventually lead to its own obsolescence.

To understand why this should occur in music, it is enough to realize that each variable imposes restrictions on the others. When Bach created a perfect balance between polyphony and functional harmony in the early 18th century, rhythmic flexibility was sacrificed. This was necessitated by the need to clarify the direction of harmonic progressions. Tonality also tended to restrict the subtle freedom of modal melodies, an important characteristic of Renaissance and medieval music. As an example of the changes that occurred, two excerpts from the Gloria section of the choral mass will be highlighted. At the height of the Counter Reformation, Palestrina's music defined the summit of modal contrapuntal composition The ethereal beauty of his Pope Marcellus Mass is reflected in the short but beautiful setting of the words "Cum Sancto Spiritu" which end the Gloria section.

(S36) : http://www.culturalmath.com/media/Sound-36.mp3

One hundred years later, the setting of these words are found in the monumental B Minor Mass by Johann Sebastian Bach.

(S37) : http://www.culturalmath.com/media/Sound-37.mp3

This rendition reflects the earthy rhythmic spirit of the baroque period and its mastery of the harmonic and contrapuntal implications of tonality.

For another 200 years, rhythm and melodic phrasing would be subordinate to harmonic practice. By the end of the 19th century, however, composers like Wagner (1813-1883)

(L1) http://en.wikipedia.org/wiki/Richard_Wagner

and Debussy (1862-1918)

(L2) http://en.wikipedia.org/wiki/Claude_Debussy

attempted to escape the rhythmic strictures of functional harmony. Wagner achieved this by the increasing ambiguity of chromaticism in his music. Debussy, on the other hand, employed some of the modal elements of the earlier periods and also composed with exotic scales in which the interval of the tritone figures prominently (refer to the table of intervals in Chapter 3). He was able to create a musical counterpart to the Impressionist style of the visual arts. Where the Impressionists succeeded in capturing a fleeting moment in the ever changing visual environment, Debussy admirably evoked the random impressions of nature in motion. In his tone poems like La Mer (L3) and Nuages (L4), we are enthralled by the movement of clouds and the sea. In many of his works he purposely blurred the traditional cyclical pulse of rhythmical repetition in order to cast the listener on a drifting sea of indeterminate sound movement.

L3: http://en.wikipedia.org/wiki/La_Mer_(Debussy)

L4: http://en.wikipedia.org/wiki/Nocturnes

(S38) : http://www.culturalmath.com/media/Sound-38.mp3

Ever since tonality emerged as the accepted style of music, theorists have attempted to make a case for its universality as a natural phenomenon. In other words, so this argument goes, given sufficient time, every culture would eventually evolve functional harmony as a natural consequence of the physical relationships between tones and the psycho-physiological nature of human hearing. In his pioneering work on tonality, Rameau (1683-1764) (L5) http://en.wikipedia.org/wiki/Rameau seized upon the overtone series as an explanation for harmonic practice. (Figures 1 & 2)

Figure 1

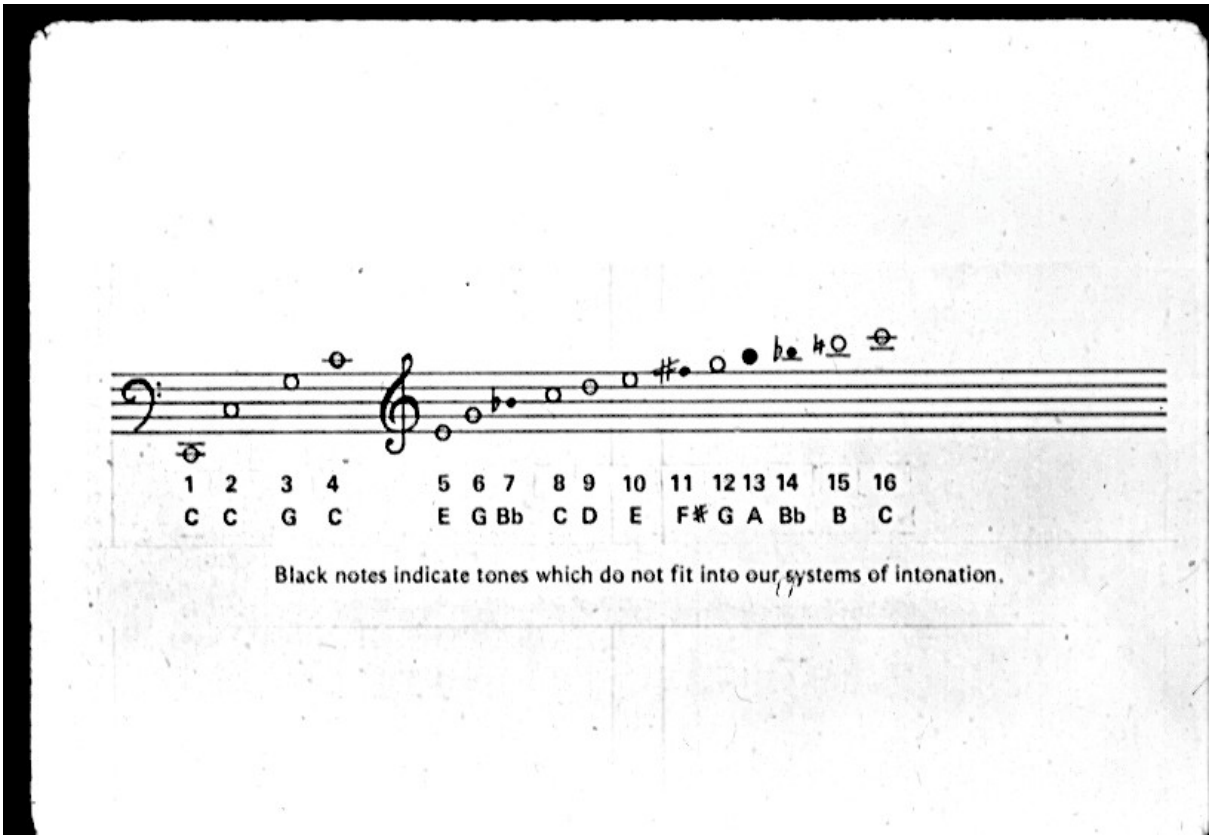
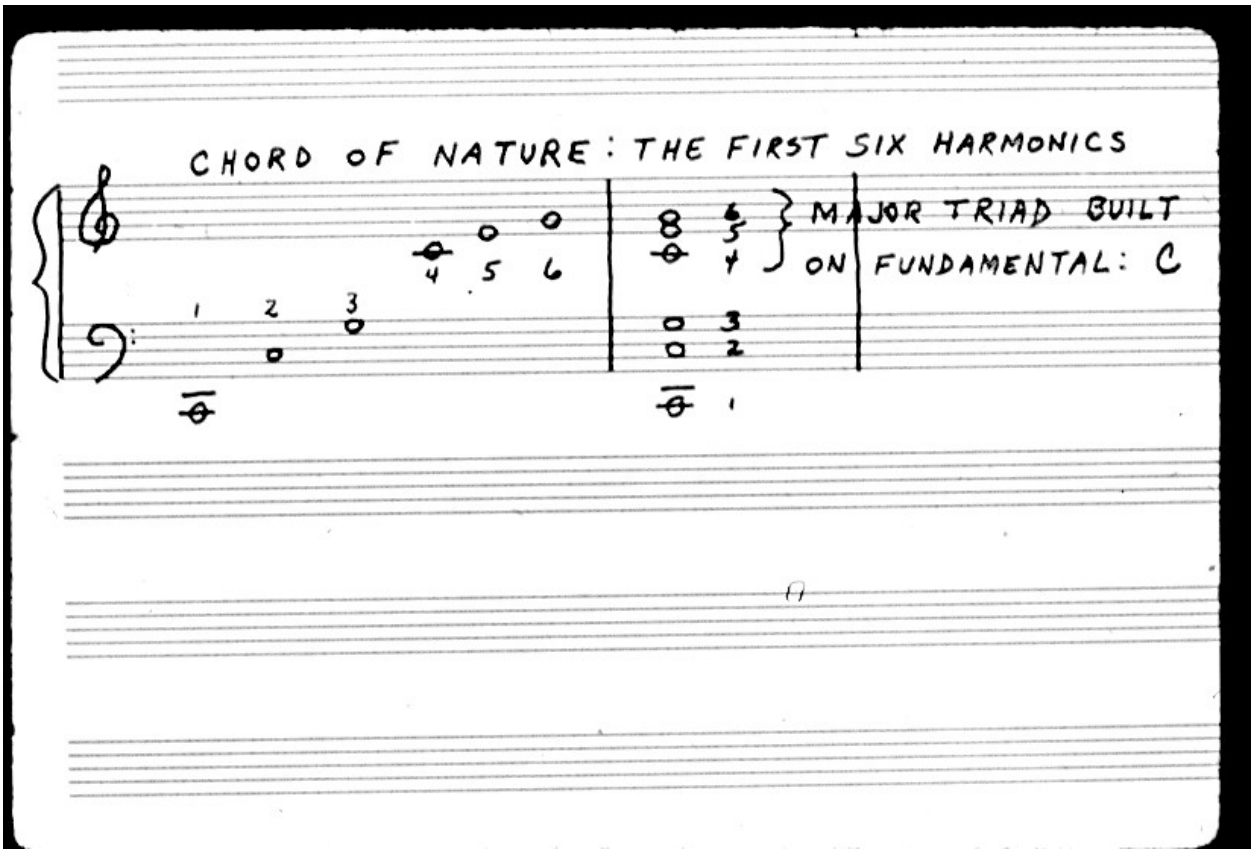


Figure 2


CHORD OF NATURE: THE FIRST SIX HARMONICS
1 2 3
4 5 6
MAJOR TRIAD BUILT
ON FUNDAMENTAL: C

He and other theorists were impressed by the fact that the major triad, the so-called "chord of nature" appeared within the first six harmonics of any tone produced, and that this chord gave the strongest harmonic definition of a Tonic root. (L6) http://en.wikipedia.org/wiki/Overtones

Unfortunately, explanations of harmonic practice based upon the overtone series have been validly criticized for leaving too many questions unanswered. For example, the minor triad also defines a Tonic just as strongly as the major triad, but it is not to be found in the overtones series.

More recently attention has been focused on another acoustical phenomenon, the combination tones.
(L7) http://en.wikipedia.org/wiki/Combination_tone
These are summation and difference tones that can be heard when tones are produced simultaneously. For example, if a tone of 180 cps is played with a tone of 60 cps, the tones 120 cps and 240 cps will also be produced (180 - 60 = 120 ; 180 + 60 = 240). The existence of difference tones was reported by the Italian violinist, Tartini (1692-1770), who produced them by double stopping on the violin in 1714. Since then the existence of difference and summation tones has been verified scientifically. Helmholtz (1821-1894)
(L8) http://en.wikipedia.org/wiki/Helmholtz#Acoustics_and_aesthetics
and others have shown that the asymmetrical structure of the inner ear contributes to our hearing these combination tones.

In 1937, the composer-theorist, Paul Hindemith (1895-1963),
(L9) http://en.wikipedia.org/wiki/Hindemith#Hindemith.27s_musical_system
integrated both the overtones and combination tones in his own theory of harmonic fluctuation. Although his theory contributed to ordering his own style of dissonant counterpoint, it is looked upon today as a throwback, an

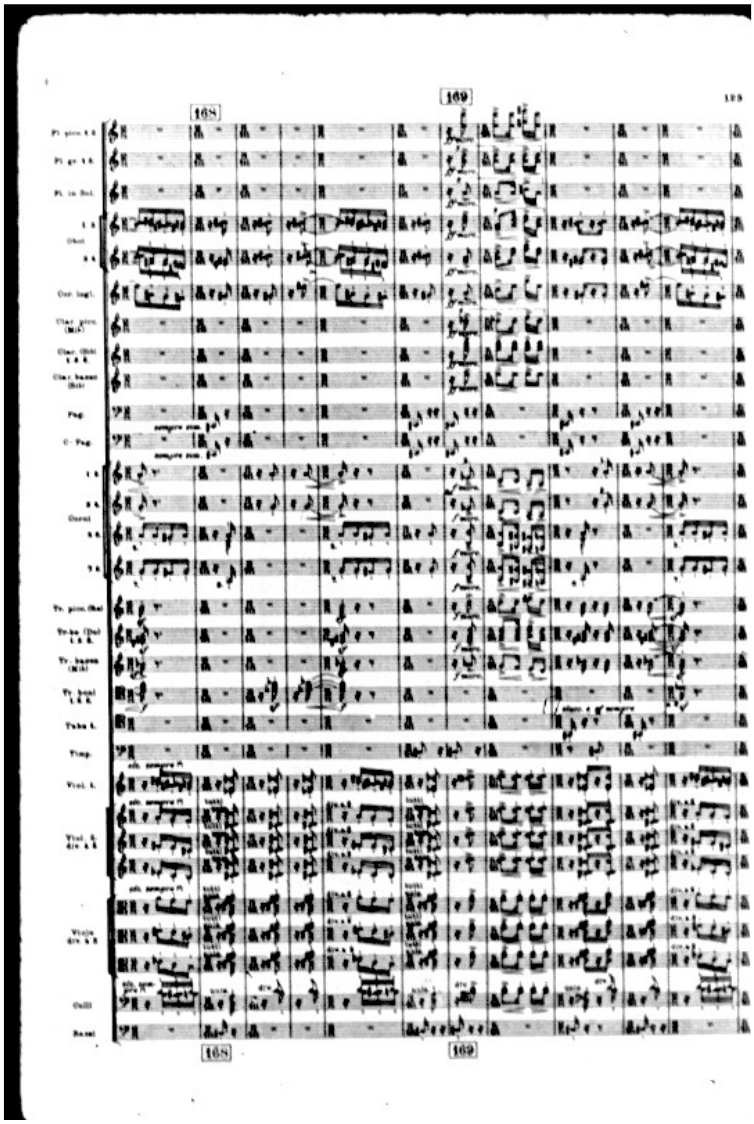
attempt to preserve the stylistic elements of tonality and modal counterpoint. Hindemith found his source of inspiration in medieval music and it is no wonder that he was one of the finest contrapuntalists of his time.
A typical example of this characteristic of his work is captured in the opening movement of the Kammermusik Op. 24 No. 2
(S39) : http://www.culturalmath.com/media/Sound-39.mp3
There is an element of Pythagorean mysticism in his works. He was fascinated by medieval ideas about music and even wrote a symphony (The Harmony of the World) based upon Kepler's thoughts about the music of the spheres. (L10) http://en.wikipedia.org/wiki/Johannes_Kepler

The work, however, that jolted the musical world into the 20th century was written by the Russian-born composer, Igor Stravinsky (1882-1971) (L11) http://en.wikipedia.org/wiki/Stravinsky
(S40) : http://www.culturalmath.com/media/Sound-40.mp3

The title of the composition is The Rite of Spring and it received its first performance as a ballet in Paris, 1913. The choreography depicted fertility rites of primitive Russia and was so angrily received by the Parisian audience that a riot ensued at the performance. It turned out to be one of the most momentous and physically violent occasions in the history of music. The barbarity of the music was matched by the ferocious reaction of the audience. How prophetic that this artistic expression of primitive ceremonial rites and violent sacrifice would one year later be amplified on such an enormous scale on the battlefields of France. It is no mere coincidence that the political and artistic revolutions of our era all commenced about the same time.

Stravinsky's rejection of Romanticism was most notably evidenced in his treatment of rhythm. It has been pointed out that functional harmony re-

quired a simple rhythmic sub-structure to enable the melodic flow to co-ordinate smoothly with harmonic progressions. Duple and triple metrical patterns dominated musical practice for 300 years. Rhythmic regularity was essential for focusing interest on melodic and harmonic invention. Stravinsky challenged this by introducing unrelated meters which alternated from measure to measure, completely eliminating the feeling of cyclical repetition of beat patterns in certain passages. (Figure 3) is a typical page of the score for The Rite of Spring that reveals how frequently the meters change almost from measure to measure. Imagine how the dancers must have felt in learning their steps to this complex score.



Through the use of highly dissonant sound combinations, syncopation, and additive rhythmical patterns derived from Russian folk music, he administered, as Aaron Copland so aptly described, a "rhythmic hypodermic" to the music of western civilization.

This emphasis on the folk derivations of additive rhythms was carried forward into the 1920's by the great Hungarian composer, Bela Bartok (1881-1945) (L12) http://en.wikipedia.org/wiki/Bartok_Bela
A brief example from his fourth string quartet captures this impetus.
(S41) : http://www.culturalmath.com/media/Sound-41.mp3

Bartok is also reputed to have used the Golden Section in structuring the forms of certain of his compositions. The form of the first movement of his Music for Strings, Percussion, and Celesta, is a prime example of his interest in this proportion.
(L13 &L14) http://en.wikipedia.org/wiki/Golden_ratio
http://en.wikipedia.org/wiki/Golden_ratio#Music

Another aspect of this rhythmic revolution was the absorption of jazz elements into every level of musical expression during the 1920's.
Listen to this remarkable historical performance.
(S42) : http://www.culturalmath.com/media/Sound-42.mp3

In contrast to the irregular patterns of The Rite of Spring, jazz requires an undercurrent of regular beats to create its syncopated style.
(L15) http://en.wikipedia.org/wiki/Jazz
Today, jazz and its derivatives have become an international form of musical expression. Almost every important composer of the 20th century experimented with jazz forms. However, with a few notable exceptions, it has not been exploited extensively by symphonic composers. The reasons for this

are obvious when one examines the restricted tonal idiom of the jazz style. The very elements which make it attractive as a popular style, work against it in the extended developments which characterize symphonic and chamber music forms.

A composer working with the blues, for example, is immediately confronted with the implications of its cultural specificity. As soon as this style is taken out of its normal musical context and treated in the formal setting of a large scale composition, it loses much of its natural expressive impact.

Another limitation of jazz is its dependence upon 19th century Romantic harmony, despite the innovations it has harmonically introduced into that style. When jazz emerged in the 1920's as a musical force to be reckoned with, composers had already broken away from the system of functional harmony, advancing new systems of tonal organization. So, even when they experimented with the novel elements of jazz, it was integrated into their new tonal vocabularies.

Stravinsky, for example, wrote a Ragtime,

(S43) : http://www.culturalmath.com/media/Sound-43.mp3

but except for certain rhythmic and melodic gestures, its style shares little in common with the popular music of that time.

In particular his works contained harmonic elements that extended 19th century harmonic practice. This style which involves the simultaneous presentation of different keys or the rapid alternation between different keys was named "polytonality." (L16) http://en.wikipedia.org/wiki/Polytonality Aurally, it produced an effect analogous to the simultaneous perspective views of the cubists.

Besides Stravinsky, composers like (L17) Milhaud (1892-1974), Honeggar (1892-1955) (L18), Prokofieff (1891-1953), (L19) and Shos-

takovitch (1906-1975) (L20) employed it extensively. It was particularly suitable for portraying humor and irony.

L17: http://en.wikipedia.org/wiki/Darius_Milhaud

L18: http://en.wikipedia.org/wiki/Honegger

L19: http://en.wikipedia.org/wiki/Prokofiev

L20: http://en.wikipedia.org/wiki/Shostakovich

In the Polka from Shostakovitch's ballet, The Age of Gold,
(S44) : http://www.culturalmath.com/media/Sound-44.mp3
we are able to respond to the music with delight because of our thorough orientation to the system of tonality. All of the "crazy" wrong notes that tickle our funny-bone were carefully selected by the composer as aberrations in the context of this traditional tonal system. Underlying the melodic high jinks, one can discern the basic tonic-dominant-sub-dominant structure with which we are so familiar.

The impression should not be given that poly-tonality was restricted to humorous musical contexts. There have been many fine serious compositions exploiting this system. However, despite its masterful exploitation by many of the greatest composers of this century, polytonality is now considered to be a transitional phase in the evolution of musical style. The simultaneous movement of melodic lines in different tonalities was matched by the shifting of accents created by poly-rhythms. After the first World War, scores appeared by Stravinsky (e.g. L'Histoire du Soldat) and others in which different meters were juxtaposed. Thus, while one instrument might be playing a waltz rhythm in three quarter time, another could be performing a march in four-four time. This resulted in an overlapping of conflicting

rhythms that tended to obscure the normal cadential phrasing associated with traditional tonal music. One of the pioneers in these techniques was an American original, Charles Ives (1875-1954), whose works are filled with poly-rhythmic renditions of native folk songs, patriotic songs, and hymns from his New England heritage. An excellent example of his style is the last few bars of his second symphony.

(S45) : http://www.culturalmath.com/media/Sound-45.mp3

(L21) http://en.wikipedia.org/wiki/Charles_Ives

It is important to realize that it took several decades for music of this complexity to be performed. The training of musicians had to catch up to the innovations that appeared so rapidly in the early part of the 20th century. It is now commonplace for the trained orchestral musician to handle techniques that could only be touched by a virtuoso when they were introduced.

Of all the innovations that arose to supersede functional harmony, only one system really came to grips with its mathematical foundations. Earlier, we described tonality as an ordering of the chromatic set that established one of them as a tonal center or reference level of equilibrium. In the second decade of the 20th century, the composer, Arnold Schoenberg, introduced a system that effectively dissolved this relationship and thereby created a new foundation for musical composition.

(L22) http://en.wikipedia.org/wiki/Arnold_Schoenberg

Before we examine Schoenberg's system, one might seriously question why composers should desire an alternative to the system of tonality. After all, it has enabled music to develop a variety of marvelous styles ever since it was instituted in the 17th century. There are still many excellent composers who are able to realize unique and original compositions

within its domain.

The impulse to replace tonality by a new system which denied its structural foundations arose through a belief that it had exhausted its expressive possibilities. Wagner had extended tonality to its ultimate limits. As Erickson points out, "There are so many altered chords and so much of the melodic motion is chromatic in Wagner's latest works, that the essential root relations which guarantee the 'keyishness' are almost completely obscured ... Wagner's kind of tonality, with its emphasis on harmonic ambiguity, tonal expansion, chord color and chord intensity, mirrors his aesthetic goals, boundlessness, continuous expansion, emotionalism." **1**

The most typical example of this description is the Prelude to Wagner's opera, Tristan and Isolde

(S46) : http://www.culturalmath.com/media/Sound-46.mp3

(L23) http://en.wikipedia.org/wiki/Tristan_und_Isolde

The theme develops in great emotional swells that continually elude any clear definition of a Tonic goal. This continues from the prelude to the end of the opera, some four hours later, when at last the composer finally resolves the harmonic flow in the famous "Liebestod" (Love-Death).

Schoenberg began his composing career as a true descendant of Wagner's tonal style. Yet, even in his early Romantic compositions (e.g. Verklarte Nacht) (L24) http://en.wikipedia.org/wiki/Verklarte_Nacht

(S47) : http://www.culturalmath.com/media/Sound-47.mp3

there is a hint of his unique interest in the independence of melodic lines. There is a feeling that his melodic inspiration was already trying to break out of the web of chromatic harmonies which held it in check. For Schoenberg, this restless chromaticism had nowhere to evolve unless it were replaced by a new system which permitted a greater melodic and rhythmic

freedom. He experimented for about two decades before arriving at a logical foundation to control the contrapuntal elements of his style.

There is no question that Schoenberg belongs to the Pythagorean tradition as it developed in music. He believed that a rational order was necessary for the development of tonal resources. In upholding this belief, he laid the foundation for a new language of musical expression.

The essentials of the system are easy to describe. Imagine that a new game has been invented. You are given twelve empty boxes and twelve different playing pieces. (Figure 4) Each of the pieces corresponds to a different tone that is found in one octave of the chromatic scale. The rules of the game are simple. The boxes are lined up in a row and the player selects one playing piece at a time to place in each of the boxes. When this task is completed you will have composed what is called a "tone row." A mathematical question to be asked at this point is: How many tone rows can be constructed by this process? Those who have studied the algebra of permutations will remember this as a counting problem. The first task is to select one piece out of the 12 available to put in the first box.

Figure 4

Once that is completed, one piece out of the remaining 11 is selected for the second box. The process continues, successively dwindling the supply of pieces, until there is only one piece left for the last box. The number of tone rows that can theoretically be obtained in this manner is given by the mathematical symbol, 12! (read as "12 factorial").

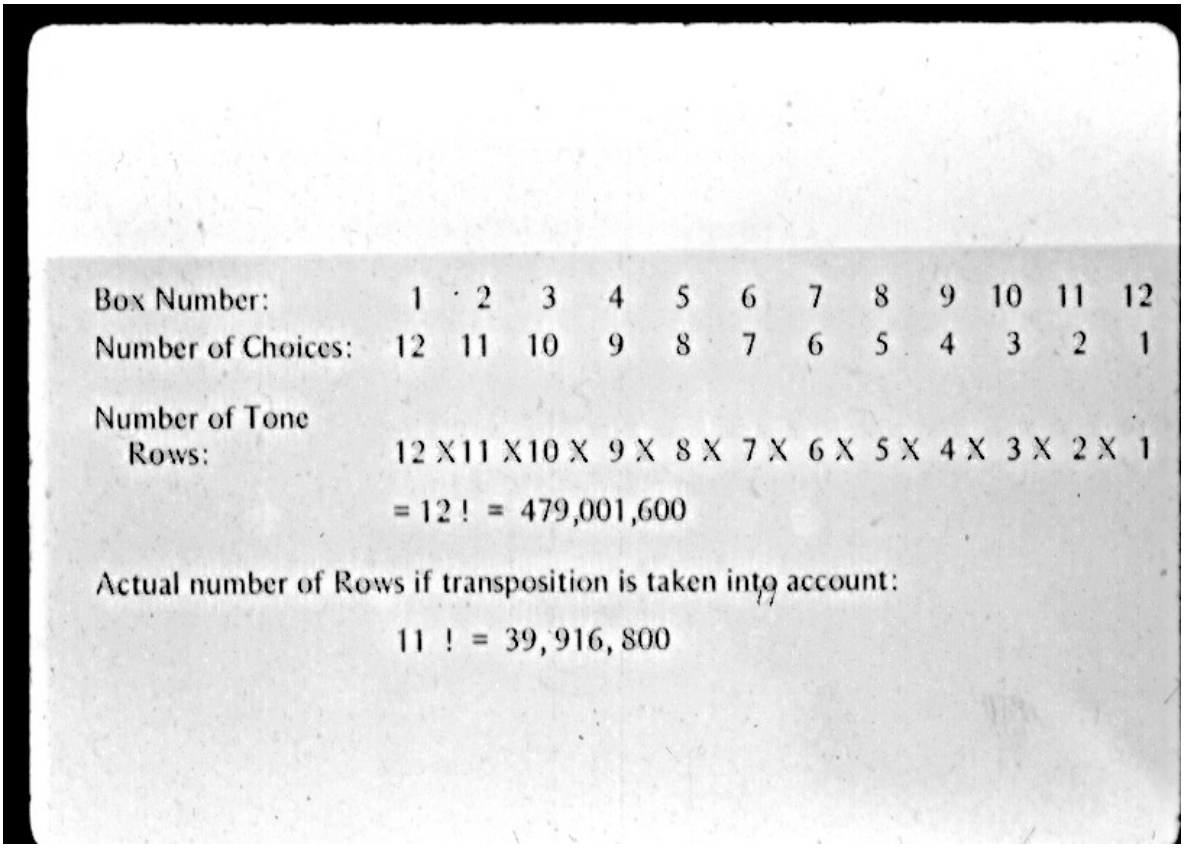
12! = 12 X 11 X 10 X 9 X ... X 3 X 2 X 1 = 479,001,600.

Hence, a considerable number of tone rows can be obtained in this manner. If we were to compose tone rows in this way, we would be basically following the system devised by Schoenberg. In actuality, the system is a bit more complicated, as composers who follow it do not concern themselves with the actual tones so much as with the intervals which successively occur between them. If the interval relationships are held constant, transposition of the whole row may be carried out starting with any arbitrary tone without destroying the integrity of the row. This cuts the number of available rows to 11! = 39,916,800. From this point of view it is more accurate to speak of an interval row rather than a tonal row. (Figure 5)

Since music occurs sequentially, certain rules have been established to preserve the style of music that is based upon this system. The row can be ordered as written or played in reverse or as it is called, retrograde order. Also, all of the intervals between the tones can be replaced by their inversions (Figure 6) (i.e. an ascending fourth by a descending fourth). The inversion row can also be sounded in retrograde order. All told, there are four basic versions of the tone row. (Figure 7) illustrates theses forms as they occur in Schoenberg's Fourth String Quartet.

(S48) : http://www.culturalmath.com/media/Sound-48.mp3

Figure 5



Box Number:	1	2	3	4	5	6	7	8	9	10	11	12
Number of Choices:	12	11	10	9	8	7	6	5	4	3	2	1

Number of Tone Rows: $12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

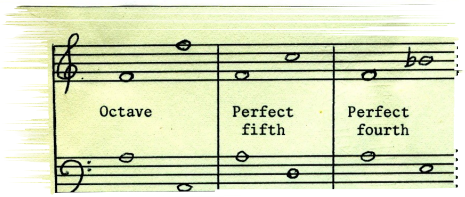
$= 12! = 479{,}001{,}600$

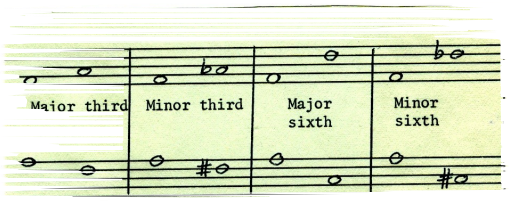
Actual number of Rows if transposition is taken into account:

$11! = 39{,}916{,}800$

Figure 6

Melodic intervals and their inversions





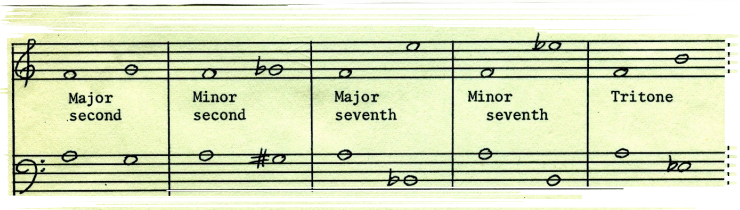


Figure 7

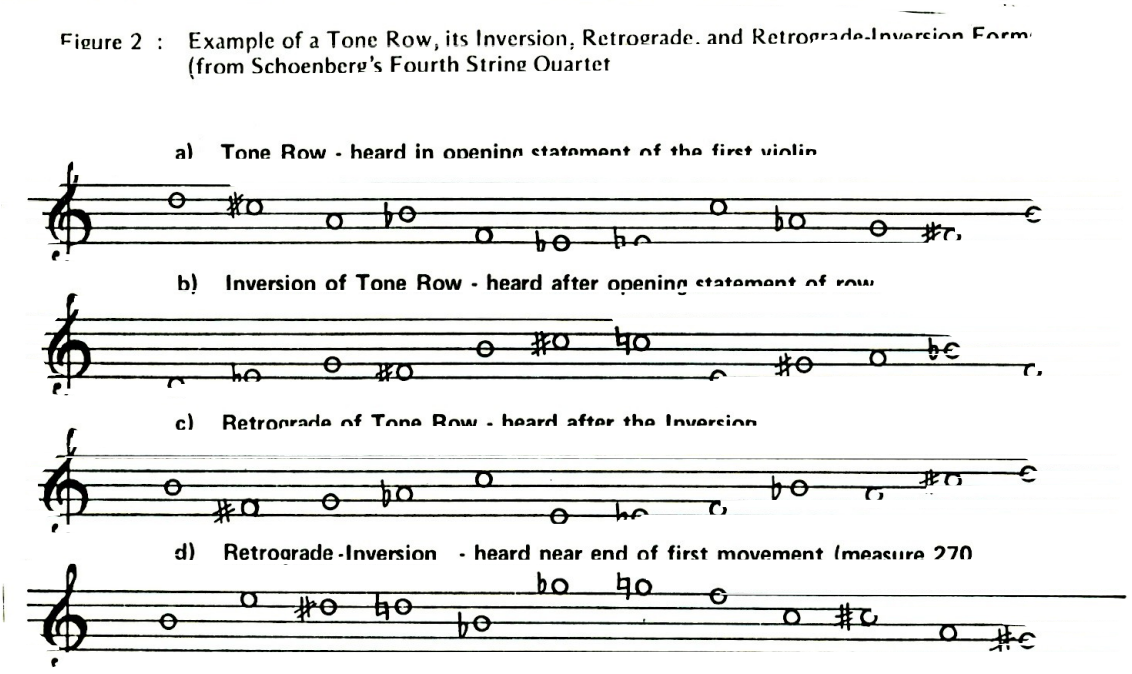


Figure 2 : Example of a Tone Row, its Inversion, Retrograde, and Retrograde-Inversion Form
(from Schoenberg's Fourth String Quartet
a) Tone Row - heard in opening statement of the first violin
b) Inversion of Tone Row - heard after opening statement of row
c) Retrograde of Tone Row - heard after the Inversion
d) Retrograde-Inversion - heard near end of first movement (measure 270

Another rule employed is that no matter which of the four versions of the row is selected, it must be used in its entirety before being repeated or replaced by one of the other versions.

Additional variety is created by the use of complementary intervals , so that an ascending major third could be replaced in the series by a descending minor sixth. In addition the basic intervals of the series can be expanded by octave transpositions.

When listening to music composed in this system, one is struck with its intensity. It is highly erratic, changeable, and dissonant. It would be difficult to remember a distinctive melody, though it is possible with practice and concentration. The cues we are used to picking up in listening seem to be absent for the most part on initial hearing of this music.

This lack of orientation is primarily due to the negation of functional harmony. Without the basic underpinning of melodic motion by the familiar tonic-dominant-sub-dominant relationships, we can no longer rely upon the traditional directions that have oriented our listening. Instead of a system of tonality where a single tone can function as a level of equilibrium, Schoenberg's system projects us into a pan-tonal world where home base is an interval pattern, one of the nearly 40 million that we learned were possible. The result is an independence between tones that neutralizes harmonic tendencies, at least as we know them. It is this neutrality or lack of differentiation that makes the system so adaptable to individual expression.

Composers like Berg, Webern, and Stravinsky have demonstrated its flexibility in applying it to their own personal styles. They have been able to do this because their personal styles are identifiable by the treatment they accord the other variables of music such as rhythm, texture, and dynamics, to name a few. For example, it was not until the 1950's that Stravinsky com-

posed in the twelve tone system (as Schoenberg's system is frequently called), but anyone familiar with Stravinsky's unique treatment of rhythm would not mistake it for the music of any other composer.

The escape from tonality has had its price. It has taken over fifty years to develop even a small appreciative audience for twelve tone music. The most popular works in this style are by Alban Berg (1885-1935)
(L25) http://en.wikipedia.org/wiki/Alban_Berg
who was able to adapt the system to his own natural lyricism. One of the most important operas of the 20th century has been Berg's Wozzeck which admirably sets an expressionistic plot in the highly charged dissonant style of the twelve tone system. To bridge the gap between this new style and the music of the past, Berg based the structure of the various scenes of the opera on traditional musical forms; i.e., the suite, rhapsody, passacaglia, sonata. (L26) http://en.wikipedia.org/wiki/Wozzeck

After the Second World War, it was the music of another of Schoenberg's students, Anton Webern (1883-1945)
(L27) http://en.wikipedia.org/wiki/Webern
that was to have the most profound influence on the post-war generation of composers. In a variety of extremely short, concise works, Webern concentrated the twelve tone system into exploring the variable of individual tone color. In a kind of musical analogue to Seurat's pointillist technique in painting, the tone row and its variants were distributed tone by tone to the different instruments that were carefully selected for their textural contrasts. This atomization process explains the brevity of his compositions. The listener is forced to concentrate on the individual tonal events rather than an overall flow of motivic development. This results in a high degree of compression in the aural information received. Webern's acute sense of proportion al-

lowed him to carry off this revolution in tone color and durational values with consummate artistry. His works are among the most highly ordered to be found in the music up to that time. It led in the direction of a total "serialization" of all of the variables of music.

(S49) : http://www.culturalmath.com/media/Sound-49.mp3

Serial music represents an ultimate mathematical formulation of the variables of music into discrete events. The composer who is generally acknowledged as a key figure in the development of this style is Olivier Messiaen (1908-1992) (L28) http://en.wikipedia.org/wiki/Messiaen

He extended the twelve tone system to cover durations, dynamics, and timbre (tone color produced by method of attack). For example, Stuckenschmidt writes of one of Messiaen's piano compositions: "(it) is strictly constructed from the following material:

1 Melodic series of 36 notes

2 Rhythmic series of 24 durations

3 Dynamic series of 7 intensities

4 Timbre series of 7 modes of attack

Each of the 36 notes of the melodic series has an unchanging duration, intensity, and mode of attack. This four-fold determination is then set in three-part canon." **2**

The human limitations encountered in performing music of this exactitude was a strong encouragement to composers in turning to electronic techniques for the realization of their serial conceptions. Beginning with the 1950's, the emergence of the tape recorder, the electronic synthesizer, and the computer finally made available the technology for a complete serialization of the elements of music. It has placed in the hands of the composer

the capability of producing and organizing any conceivable combination of sounds.

Parenthetically, it should be pointed out that this technological revolution would not have been possible without the mathematical foundation developed by Fourier (L29) http://en.wikipedia.org/wiki/Joseph_Fourier and Maxwell (L30) http://en.wikipedia.org/wiki/James_Clerk_Maxwell in the 19th century. Fourier's series governing harmonic analysis and Maxwell's equations for the behavior of electromagnetic radiation are fundamental to electronic sound synthesis.

In addition to the traditional sounds of music, the natural sounds of our environment can be recorded and transformed into all kinds of different sound impressions by electronic means. No longer restricted to the sounds produced by musical instruments, noise has become a component that may be exploited in compositions. We can no longer speak of musical composition in this context. Perhaps a better term for works employing the new technology would be "sound compositions."

Sound composers run the gamut from those who impose a high level of order on the sequence of events in their compositions to those who attempt to achieve complete randomness. The inspiration for the latter approach is the improvisatory style of contemporary jazz as well as influences from Asian music. Composers like John Cage (1912-1992) (L31) http://en.wikipedia.org/wiki/John_Cage have prepared aural experiences which simulate in a concert setting the random way that sounds impinge upon our consciousness in our everyday urban existence.

In terms of information theory, a recent branch of mathematics, these aural experiences represent a tendency toward the state of maximum entropy where every sound, either musical or noise, is equally probable. Cage generated interest in his works among those who are mystically oriented by his employment of the ancient Chinese treatise, the I Ching (or the Book of Changes). He used this book and other techniques to introduce chance elements into his compositions. This statistical approach to music has gained considerable momentum recently as a result of the expanding capabilities of high speed electronic computers. The technology and theory of operating these machines has so advanced that they can now be programmed to synthesize compositions in any style, past or present. Computers have been hooked up to sound producing mechanisms, thereby eliminating the necessity of having to transcribe the compositions for live performers. The growth of this technology over the past decades has been phenomenal, thanks to the invention of solid state circuitry, a magnificent result of applied mathematics. Composers are being trained to handle this great variety of electronic resources and it is likely that the listening public will be exposed to more and more of these sound compositions.

L32: http://en.wikipedia.org/wiki/Computer_music

(S50) : http://www.culturalmath.com/media/Sound-50.mp3

Each new technological development in music has brought about an expansion of the tonal vocabulary. The refinement of musical instruments led to the establishment of the symphony orchestra in the 18th century. The mathematics of equal temperament allowed for the standardization of keyboard instruments and permitted music to be written which freely modulated to any key desired. The electronic revolution of our own time has not only made music available to everyone at the push of a button, it also

opened up a new world of possibilities in the art of musical composition. It is too early to assess the artistic validity of the experiments now being conducted in the field. As someone tartly remarked, computer music may only be of interest to other computers. Yet, there seems to be little doubt that eventually these new resources will be integrated into the uniquely human form of communication we call the art of music.

The basic problem confronting composers in any age is how to maintain the integrity of their own personal expression. The problem is particularly acute today when so many of the traditional landmarks have been washed away in the flood of new technical means for realizing a compositional idea. The tools of mathematics and technology have bestowed complete freedom of choice on the composer and, at the same time, a terrible responsibility in the making of decisions about using these tools. The phrase "freedom is not a license for chaos" weighs heavily on the creative artists of today. Mathematics is completely neutral on this point. We have seen how it can contribute to the strictest formalism on the one hand, and on the other, lead to the structuring of random musical "happenings."

Caught up in the power of number-generated patterns, the misguided composer is in danger of repressing his own musical instincts. The greatest composers of our time have understood the necessity of remaining true to their inner voices. Like Odysseus, each has bound himself with self-imposed restrictions to the mast of his own craft, finding safe passage through the Sirens' song of chaos.

CONTEMPORARY MUSIC LINKS

L1: http://en.wikipedia.org/wiki/Richard_Wagner

L2: http://en.wikipedia.org/wiki/Claude_Debussy

L3: http://en.wikipedia.org/wiki/La_Mer_(Debussy)

L4: http://en.wikipedia.org/wiki/Nocturnes

L5: http://en.wikipedia.org/wiki/Rameau

L6: http://en.wikipedia.org/wiki/Overtones

L7: http://en.wikipedia.org/wiki/Combination_tone

L8: http://en.wikipedia.org/wiki/Helmholtz#Acoustics_and_aesthetics

L9: http://en.wikipedia.org/wiki/Hindemith#Hindemith.27s_musical_system

L10: http://en.wikipedia.org/wiki/Johannes_Kepler

L11: http://en.wikipedia.org/wiki/Stravinsky

L12: http://en.wikipedia.org/wiki/Béla_Bartók

L13: http://en.wikipedia.org/wiki/Golden_ratio

L14: http://en.wikipedia.org/wiki/Golden_ratio#Music

L15: http://en.wikipedia.org/wiki/Jazz

L16: http://en.wikipedia.org/wiki/Polytonality

L17: http://en.wikipedia.org/wiki/Darius_Milhaud

L18: http://en.wikipedia.org/wiki/Honegger

L19: http://en.wikipedia.org/wiki/Prokofiev

L20: http://en.wikipedia.org/wiki/Shostakovich

L21: http://en.wikipedia.org/wiki/Charles_Ives

L22: http://en.wikipedia.org/wiki/Arnold_Schoenberg

L23: http://en.wikipedia.org/wiki/Tristan_und_Isolde

L24: http://en.wikipedia.org/wiki/Verklärte_Nacht

L25: http://en.wikipedia.org/wiki/Alban_Berg

L26: http://en.wikipedia.org/wiki/Wozzeck

L27: http://en.wikipedia.org/wiki/Webern

L28: http://en.wikipedia.org/wiki/Messiaen

L29: http://en.wikipedia.org/wiki/Joseph_Fourier

L30: http://en.wikipedia.org/wiki/James_Clerk_Maxwell

L31: http://en.wikipedia.org/wiki/John_Cage

L32: http://en.wikipedia.org/wiki/Computer_music

Mathematics and Music Bibliography

Chapter 4 - Contemporary Musical Structures

1. Erickson, The Structure of Music, New York: The Noonday Press, 1959, pp. 88-90

2. Stuckenschmidt, Twentieth Century Music, Richard Deveson, trans., New York: McGraw-Hill, 1969, p. 206

LIST OF RECORDING EXCERPTS

S36 : PALESTRINA POPE MARCELLUS MASS: CUM SANCTO SPIRITU, ROGER WAGNER CHORALE, ANGEL LP S-36022

S37 : BACH MASS IN B MINOR: CUM SANCTO SPIRITU, MONTEVERDI CHOIR, ENGLISH BAROQUE SOLOISTS, JOHN ELIOT GARDINER, COND., ARCHIV DIGITAL STEREO CD 415 514-2

S38 : DEBUSSY NOCTURNES: NUAGES, STRASBOURG PHILHARMONIC, ALAIN LOMBARD, COND. , ERATO ECD 55037

S39 : HINDEMITH KLEINE KAMMERMUSIK OP. 24, . AMSTERDAM WIND ENSEMBLE, RICCARDO CHAILLY, COND., LONDON CD SET 433 816-2

S40 : STRAVINSKY RITE OF SPRING, COLUMBIA SYMPHONY, IGOR STRAVINSKY, COND., COLUMBIA LP SET M3S 705

S41 : BARTOK STRING QUARTET NO. 4, JUILLIARD STRING QUARTET, COLUMBIA LP SET D3S 717

S42 : JOPLIN PIANO RAG, PERFORMED BY SCOTT JOPLIN TRANSCRIBED FROM PIANO ROLL, UNKNOWN SOURCE

S43 : STRAVINSKY RAGTIME ,COLUMBIA CHAMBER ENSEMBLE, IGOR STRAVINSKY, COND., COLUMBIA LP

S44 : SHOSTAKOVICH GOLDEN AGE BALLET: POLKA, PHILHARMONIA ORCHESTRA, ROBERT IRVING, COND., EMI 1959 LP TRANSCRIBED TO CD IN EMI'S FULL DIMENSIONAL SERIES

S45 : IVES SYMPHONY NO. 2 FINALE, NEW YORK PHILHARMONIC, LEONARD BERNSTEIN, COND., CBS MASTERWORKS CD MK 42407

S46 : WAGNER TRISTAN UND ISOLDE: PRELUDE, CLEVELAND ORCHESTRA, GEORGE SZELL, COND., SONY ESSENTIAL CLASSICS CD

S47 : SCHOENBERG VERKLARTE NACHT, BERLIN SOLOISTS, CD SOURCE UNKNOWN

S48 : SCHOENBERG STRING QUARTET NO. 4, FIRST MOVEMENT, KOHON STRING QUARTET, LP VOX BOX SVBX 590

S49 : WEBERN PIANO VARIATIONS OP. 27 MOVEMENT 2, PAUL JACOBS, BARCLAY CD

S50 : HILLER & ISAACSON ILIAC SUITE, UNKNOWN SOURCE