THE CULTURAL IMPACT OF MATHEMATICS

UNIT II : MATHEMATICS AND MUSIC

Chapter 1 - Acoustics

Part of the joy of attending a symphonic concert is to enter a well designed auditorium that is both visually and aurally pleasing. Both qualities should be independent of where one sits. The price of tickets is normally a function of distance from the stage. We pay more in order to see better. However, we all know through sometimes bitter experience that no matter how much care is given to the acoustical design of a hall, there will sometimes be "dead" spots. To minimize this problem and others which plague audition, there now exists a close cooperative bond between the architect and the acoustical engineer.

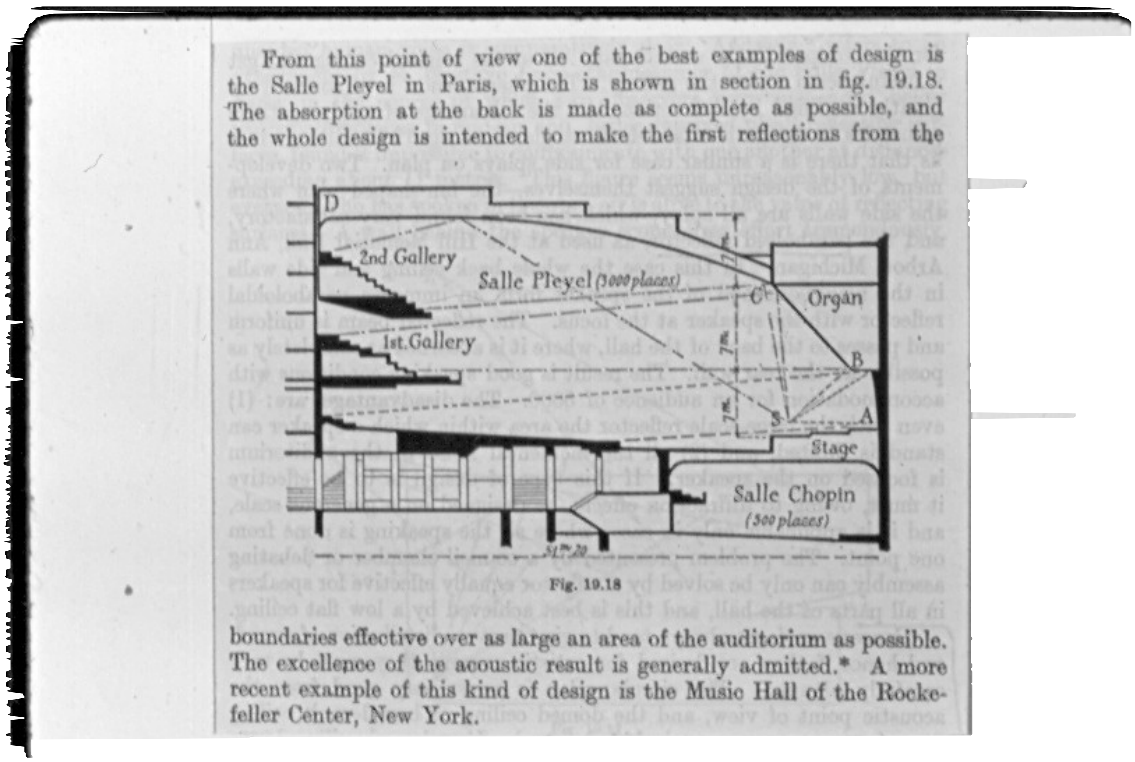
(Figure 1) shows a schematic of the Pleyel Concert Hall that is generally acknowledged to be one of the finest in the world. (L1): http://www.sallepleyel.fr/anglais/la_salle/visite_virtuelle/index.aspYet, prior to 1900 there was little scientific knowledge as to what constituted a good design for an auditorium. Since then, experiments have been conducted to isolate the variables involved in the transmission of sound waves within an enclosed structure. Generalizations have been established which are expressible in mathematical formulas and at least these are available for guidance in construction.

As we shall see these are not always fool-proof. Concert hall construction is still an elusive art and everyone involved still holds his breath until the end of the first fully attended performance. To consider some of the problems we shall begin with the production of a pure tone. (Figure 2) (S1) illustrates the vibration; of a tuning fork.

http://www.culturalmath.com/media/Sound-01.mp3

The fork vibrates at a certain fixed frequency.

Figure 1



From this point of view one of the best examples of design is the Salle Pleyel in Paris, which is shown in section in fig. 19.18. The absorption at the back is made as complete as possible, and the whole design is intended to make the first reflections from the

Fig. 19.18

boundaries effective over as large an area of the auditorium as possible. The excellence of the acoustic result is generally admitted.* A more recent example of this kind of design is the Music Hall of the Rockefeller Center, New York.

The frequency is defined as the number of complete vibrations or cycles per second. A complete cycle is generated as the prongs of the fork move to the right, left and back to the position of equilibrium or rest. These vibrations cause the air molecules surrounding the fork to compress and expand in waves. As a result, the vibrations are communicated in ever widening spheres to the air molecules further away. At normal room temperatures, this sound wave travels at a speed of 1100 feet per second.

Owing to widening distribution of energy, the intensity of the sound (that is, its loudness) diminishes inversely as the square of the distance from the source. Thus, if there were no other factors operating, a person sitting twice as far from a sound source would receive only one-fourth of its intensity. In an outdoor concert, this dissipation of energy is a critical factor. To offset this problem large shells are constructed behind the outdoor orchestra to reflect and focus the orchestra's sounds at the audience. (Figure 3).

The ancient Greeks were well versed in the reflecting properties of curved surfaces Their most successful outdoor theaters were built in the form of half-bowls and "tuned" so precisely that it is possible to hear the

Figure 2

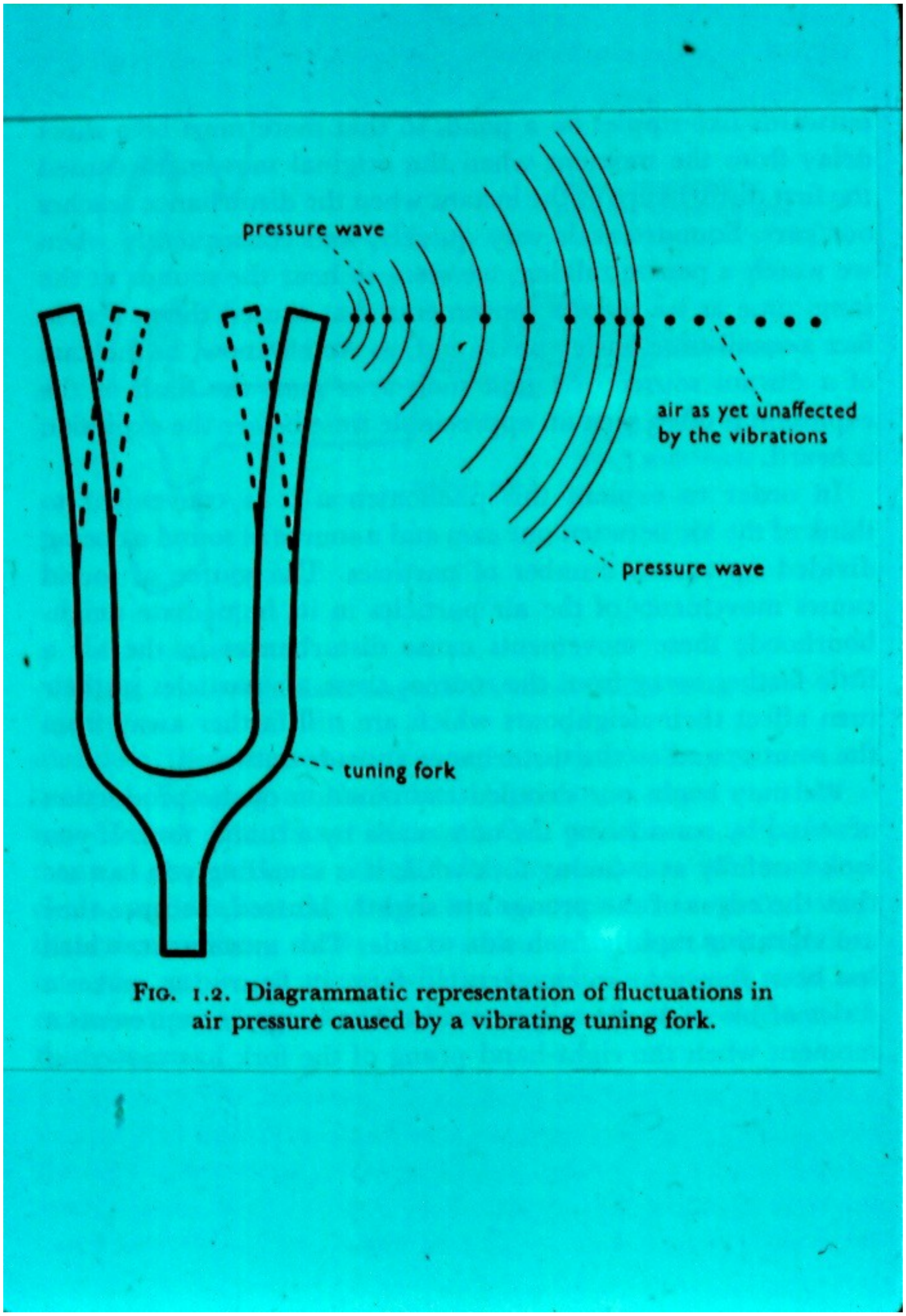
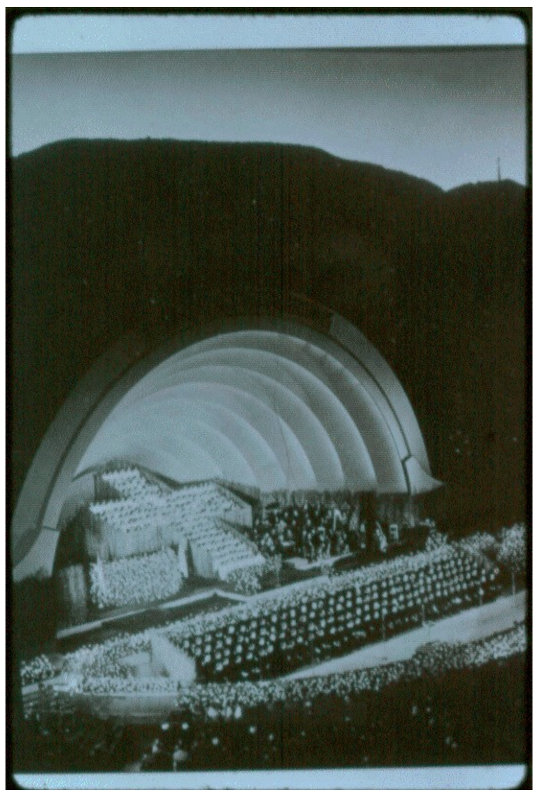


FIG. 1.2. Diagrammatic representation of fluctuations in air pressure caused by a vibrating tuning fork.

Figure 3



strike of a match from the orchestra platform while sitting on the top tier of benches.[1] (Figure 4)

Figure 4

Greek open ampitheater.

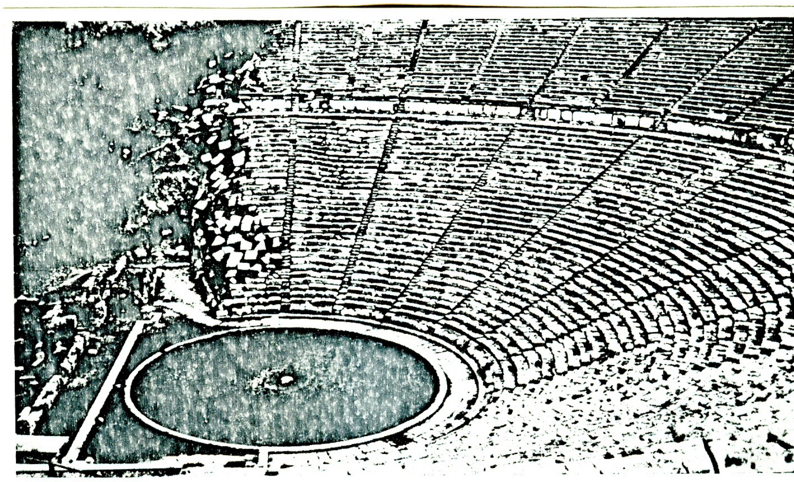
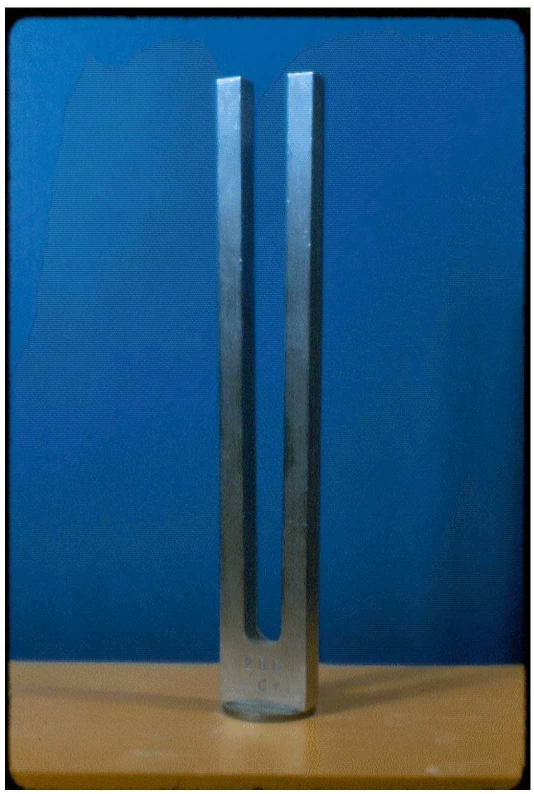


FIGURE 1. THE THEATRE AT EPIDAURUS
From a photograph by Dr. C. W. Blegen

Inside an enclosed room, the problem may be exactly reversed. To demonstrate this it is only necessary to compare the free vibrations of a tuning fork with that of a tuning fork placed to vibrate on top of an enclosed box. The sound of the latter will be greatly amplified. This is because the vibrations of the fork have been transmitted to the box and the air inside the box. What results is called resonation. It is exactly the same effect that occurs when the vibrations of the string of a violin are communicated to the sound box of the instrument. The sound is greatly amplified. (S2) (Figure 5)
http://www.culturalmath.com/media/Sound-02.mp3



Again, the Greeks understood the principle of resonation also and "..placed vases of bronze or pottery about a theater, with their open ends pointing towards the orchestra, to act as resonators. .."**2**

Similarly, in a concert hall, the walls, ceiling, and floor will receive sounds and by resonation and reflection contribute to their audibility throughout the hall. Sitting in a hall, we not only receive sounds directly from the orchestra but are continually bombarded by reflections from every direction. (Figure 6)

Figure 6

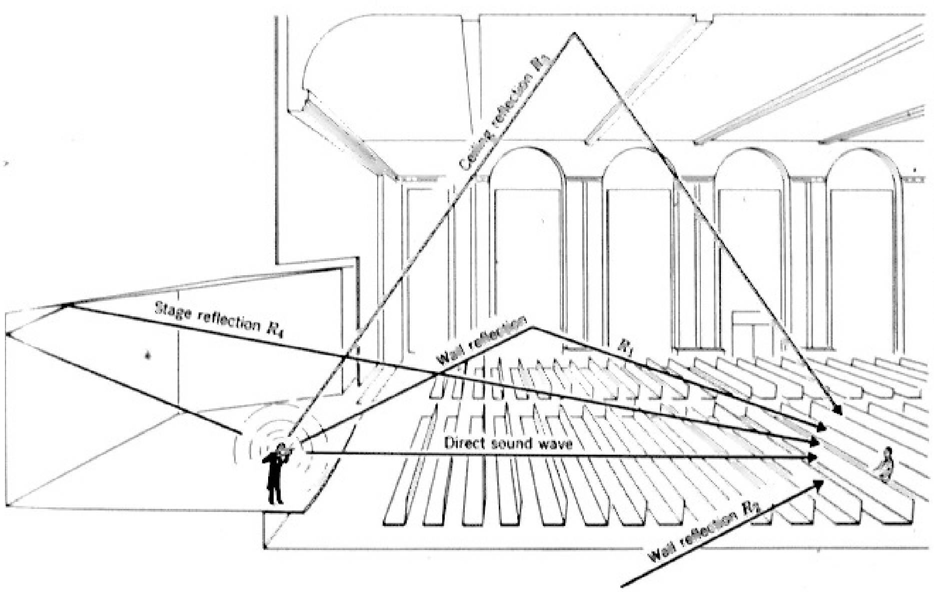


Figure 2.12. Showing the paths of direct sound and several reflected sound waves in a concert hall. Reflections also occur from balcony faces, rear wall, niches, and any other reflecting surfaces.

Another variable of acoustics is known as reverberation, which is described as “the persistence of audible sound after the source has ceased to operate.”[3] (Figure 7) shows the great conductor, Arturo Toscanini, (L2): http://en.wikipedia.org/wiki/Toscanini conducting the NBC Symphony Orchestra in the acoustically notorious Studio 8H of the NBC Broadcasting Company. Recordings made in this studio sound dry, clear, and somewhat muffled. The orchestra sounds smaller in these recordings than it does in some of the later ones made in Carnegie Hall, where the sound is fuller and more brilliant. Listen to the following recordings of Toscanini with the NBC Symphony. The first excerpt is from a live performance in Studio 8H of the 3rd movement of Haydn’s Sinfonia Concertante. The second excerpt, the 3rd movemdnt of Brahms’ Symphony No. 2 was recorded in Carnegie Hall. (S3 & S4).

Figure 7

With the NBC Symphony in Studio 8H.

The differences can be attributed to the different periods of reverberation of the two halls. Reverberation is not only important to the listener. It also has a remarkable effect on the performer who depends upon acoustical feedback as a stimulus to his performance. A pioneer in the field of architectural acoustics, W. C. Sabine, derived an empirical formula for reverberation: http://www.culturalmath.com/media/Sound-03.mp3

http://www.culturalmath.com/media/Sound-04.mp3

$T = kV/A$ where T is the time of reverberation,
V is the volume of the room,
A is the total absorption of all of the surfaces,
k = .05 if measurement is in feet, and
k = .16 if measurement is in meters.

The quantity A is determined by analyzing each surface in the room for its coefficient of absorption which varies with different material, and then using the calculus, the process of integration is performed over the surface. A range of T between 1.4 and 2.2 seconds has been found ideal for large fully occupied concert halls. For music which demands clarity of line for its effect (i.e. Mozart), the shorter period of reverberation is desirable, while for music of the Romantic period which is dependent upon lush harmonies and massive sounds, the longer reverberation is more suitable.

The architectural design is also an important factor (Figure 8) For example the new Papal Audience Hall at the Vatican built by the architect, Nervi, uses the parabolic arch to focus sound from the stage out to the audience. The reflective properties of the parabola are shown in (Figure 9). The marvel of the Audience Hall is the great expanse which is covered without evidence of supporting interior columns.

(L3) : http://en.wikipedia.org/wiki/Paul_VI_Audience_Hall

Figures 8

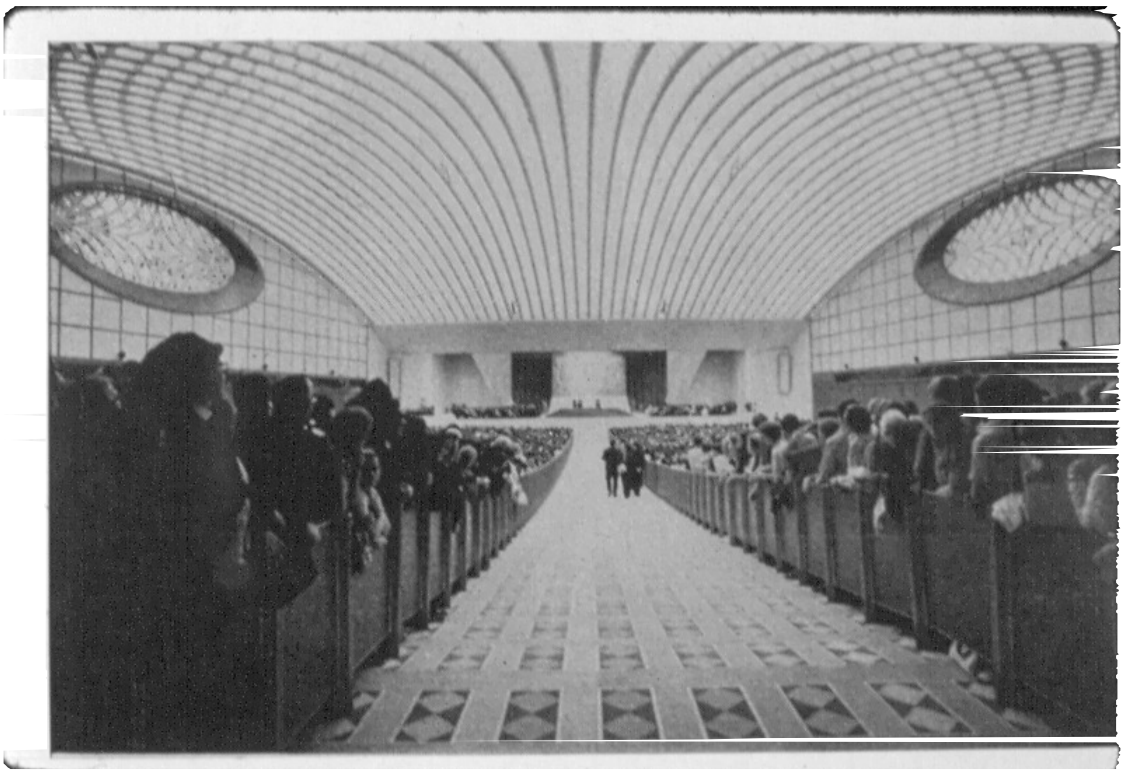
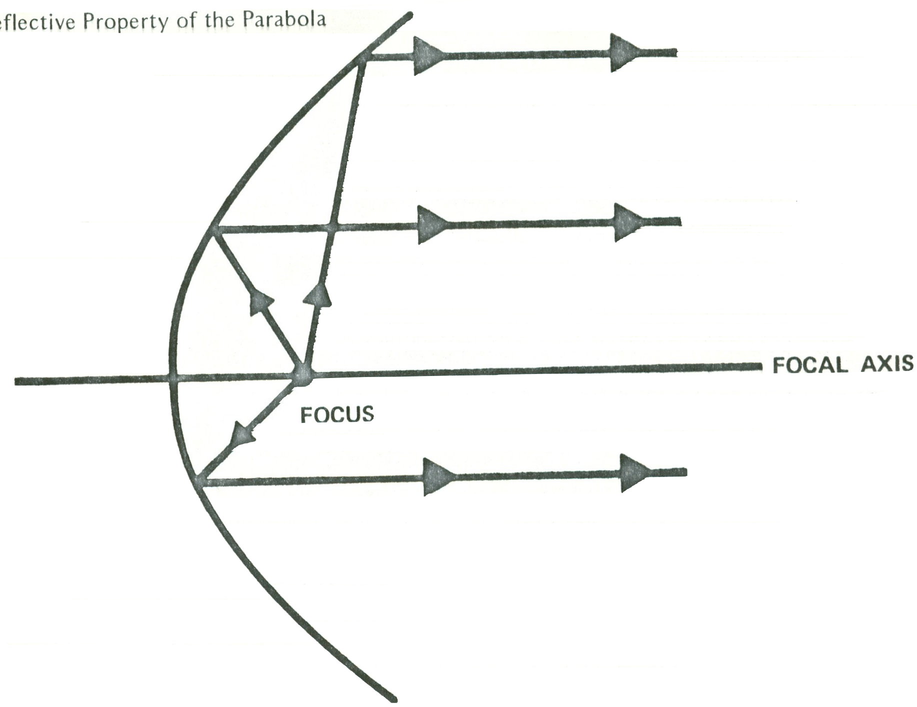


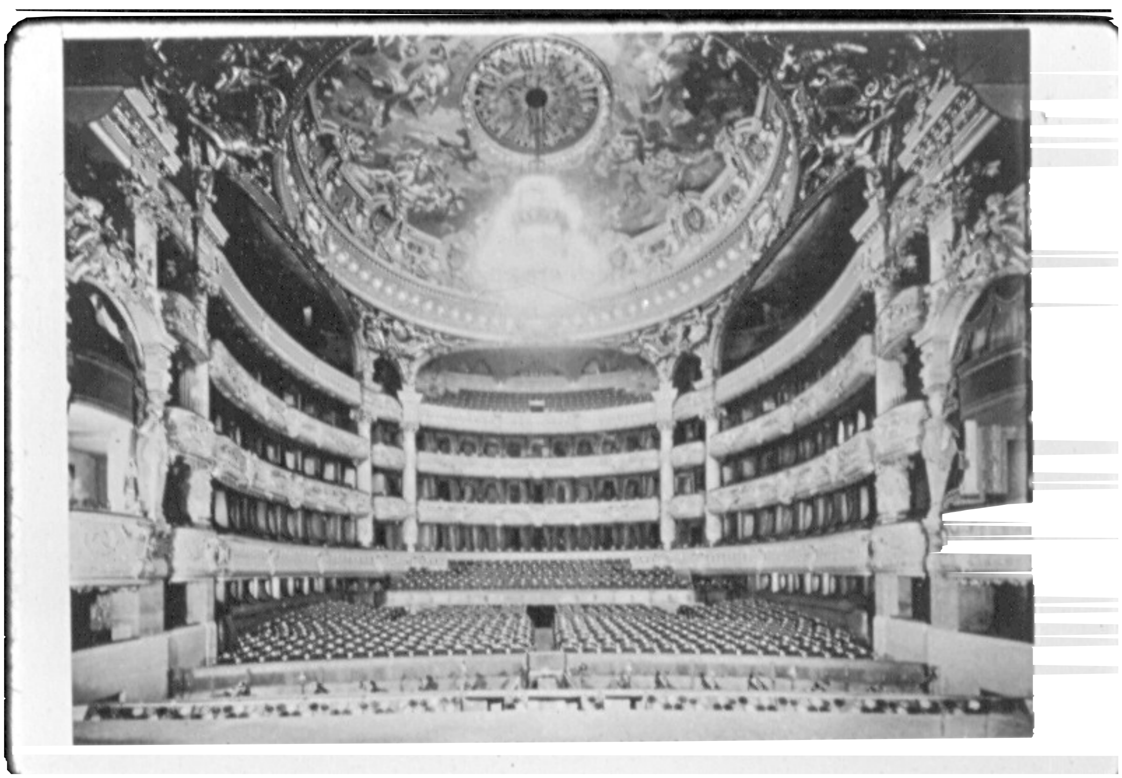
Figure 9

Reflective Property of the Parabola



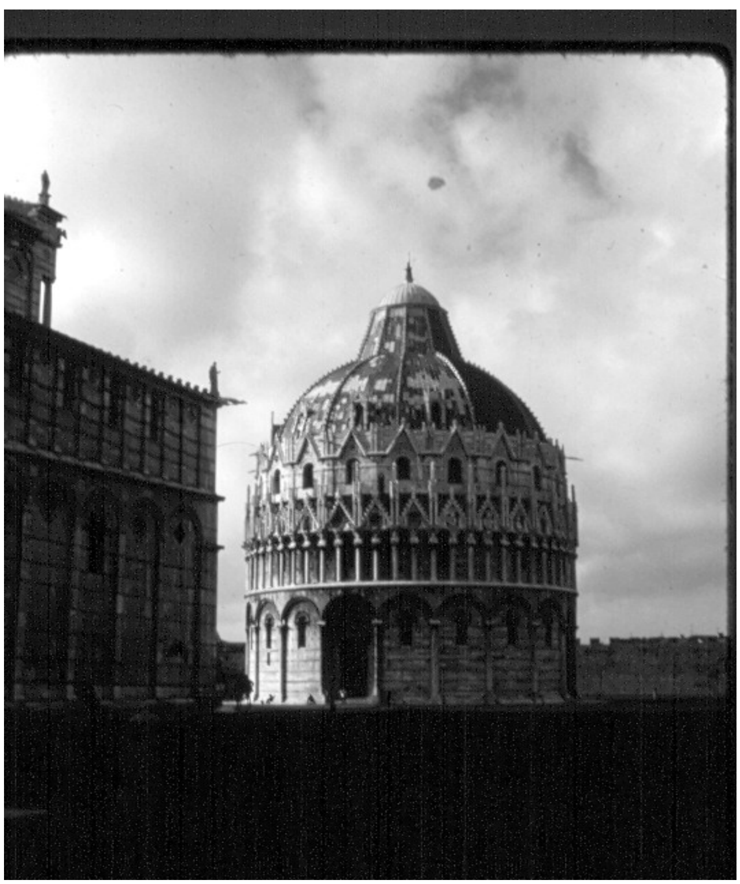
There are problems with any curved surfaces however. Care must be taken that some sounds do not build up and mask others. This is accomplished by diffusion, the breaking up of sound waves by irregular surfaces. In highly decorative halls built in the Baroque style, there is usually excellent diffusion of sound because of the exorbitant use of statuary and ornamentation (Figure 10).

Figure 10



In the Baptistry of Pisa (Figure 11), we have an example of a very reverberant building. In his excellent book, Science and Music, Sir James Jeans wrote: "Except for its windows, the interior is almost entirely of marble .... The floor is circular with a diameter of 100 feet and the roof is conical with an extreme height of 179 feet. If the interior surface were entirely of marble •.• the room would have a reverberation period of 100 seconds--sound would persist for a minute and a half. Under actual conditions, the observed reverberation period is 11 or 12 seconds. In this room, a man may sing a sequence of notes staccato and hear them combined into a chord for many seconds afterward..."[4]

Figure 11



One of the most famous or infamous, depending upon your point of view, passages in the history of acoustical engineering took place in the construction of Philharmonic Hall (now known as Avery Fisher Hall) which houses the New York Philharmonic: In these early photographs (Figure 12 &12a), the ceiling panels can be seen which could be raised and lowered to provide adjustments to the period of reverberation. In tuning the hall it was necessary to fill the seats with padded cushions to simulate the absorption effect of the audience. The ceiling panels were finally abandoned and re-placed by a conventional ceiling when it was determined that they were un-able to produce an adequate reflection of sounds coming from the stage. Unfortunately, these compromises did not solve the basic acoustical prob-lems that were inherent in the fundamental design. Eventually after years of aural anguish, the entire interior was rebuilt and the hall is now worthy to be called the home of the Philharmonic (Figure 13).
http://en.wikipedia.org/wiki/Avery_Fisher_Hall

Figure 12

Figure 15.3. Photograph of Philharmonic Hall, taken during tuning week from the rear corner of the loge.

Figure 12a and   Figure 13

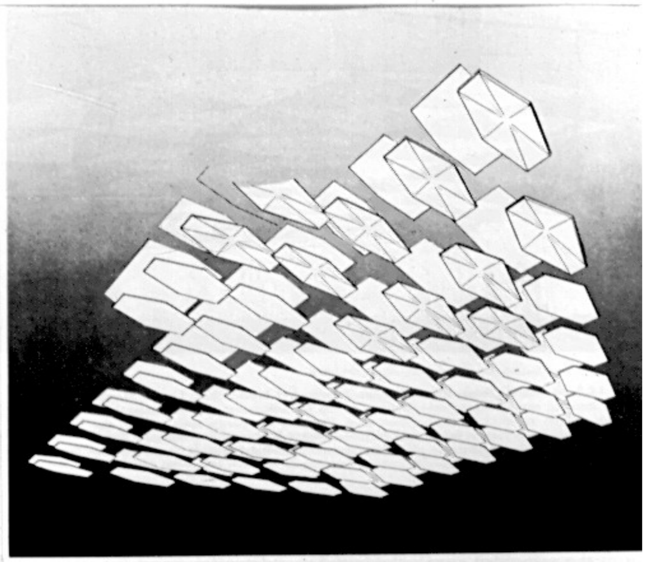
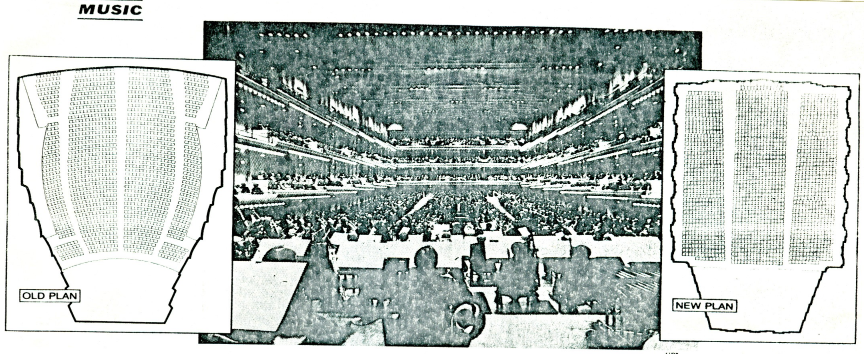


Figure 14.2. Sketch showing suspended sound-reflecting panels for achieving a short initial-time-delay gap without sacrificing reverberation time in Philharmonic Hall, Lincoln Center for the Performing Arts. Features are partly open suspended panels, high sound-reflecting ceiling, and relatively wide hall. *Architect:* Max Abramovitz of the firm Harrison and Abramovitz. *Acoustical consultant:* Bolt Beranek and Newman Inc.

Avery Fisher Hall.

MUSIC



As the players of an orchestra file on stage and take their seats, a familiar cacophony begins that marks the prelude to any concert. The players are warming up literally. This is particularly true of the wind players who produce their tones by bringing carefully designed columns of air into vibration. In order to play with correct intonation, these air columns must be brought up to playing temperature. In addition, all of the players are warming up the great variety of finely trained muscles that are needed in the perfection of instrumental technique.

The appearance of the concertmaster, the first violinist, begins the traditional and very necessary ceremony of tuning the orchestra. Wrapping his bow for silence, he then signals the oboe player to intone the standard pitch for the tone A. Internationally, this has been established at 440 cycles per second (cps). The oboist may not play that frequency exactly, but he will be very close to it. One of the reasons the oboe is selected for this role is that its pitch can vary, but very slightly. Also, it has a clear and piercing tone quality that makes it ideal as a reference standard to be heard throughout the orchestra, the woodwinds adjust their A by slight corrections in their mouthpieces while the brass players adjust the tubing leading to the valves of their instruments. (S5) (L5)

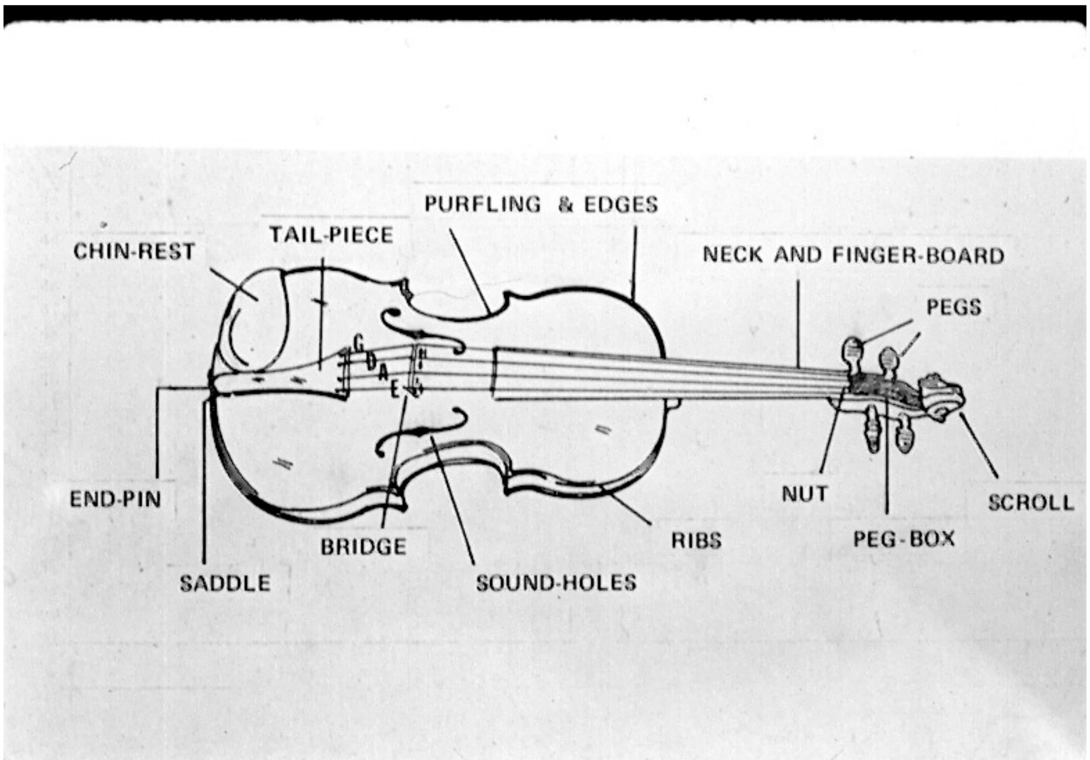
http://www.culturalmath.com/media/Sound-05.mp3

http://www.exploratorium.edu/music/movies/tuning_hi.html

The string players all have A strings that can be tuned by turning the pegs around which the strings are wrapped. On the violin, the remaining strings are tuned in the following manner (Figure 14) (S6):

http://www.culturalmath.com/media/Sound-06.mp3

Figure 14



PURFLING & EDGES
TAIL-PIECE
CHIN-REST
NECK AND FINGER-BOARD
PEGS
G
D
A
E
END-PIN
NUT
SCROLL
BRIDGE
RIBS
PEG-BOX
SADDLE
SOUND-HOLES

When the violinist tucks the instrument under his chin, the lowest sounding string, the G string, is closest to him, the E string is furthest away. Satisfied that the A string is matched to the frequency of the oboe, the violinist bows across the two middle strings, D and A, adjusting the peg for the D string until he hears the pure sound of the musical interval known as the perfect fifth. The name given to the interval will be explained shortly, but for now we shall concentrate on the ratio of the frequencies between the two tones D and A. Musicians are trained to identify the sound of this interval which is in its purest form when the frequency of the tone D is exactly 2/3 of the frequency of the tone A. If 440 cps is multiplied by 2/3, the result is 293 1/3 cps.

Next, the G and D strings will be bowed to create a perfect fifth between them. The G string is in tune when its frequency is exactly 2/3 that of the D string. Multiplying 293 1/3 by 2/3 yields 195 5/9. The actual number is unimportant to the violinist since it is impossible to count the actual number of cycles by ear. What is important is that the G and D sound harmonically identical as the D and A when they are sounded together. Finally, the highest sounding string, the E string, is tuned as a perfect fifth above the A string. The E string then has a frequency that is 3/2 times the frequency of the A string. Therefore, it vibrates at 660 cps. When all of the strings are in tune their frequencies form a geometric sequence from G, the lowest sounding string, to E, the highest sounding string. Each tone in the sequence is obtained by multiplying by the common ratio of 3/2.

Tone:	G	D	A	E
Frequency:	195 5/9	293 1/3	440	660

The sequence is independent of the actual frequencies. The only thing constant about this sequence is the common multiplier 3/2, which is the standard ratio for the perfect fifth musical interval. Pythagoras is supposed to have discovered this fact about 2500 years ago. He extended the sequence to cover seven tones in the following manner. (In modern notation, the first seven letters of the alphabet are used to designate musical tones. In the sequence of fifths however, they do not appear in alphabetical order.

Tone:	F	C	G	D	A	E	B
Frequency:	86.9	130.4	195.6	293.3	440	660	990

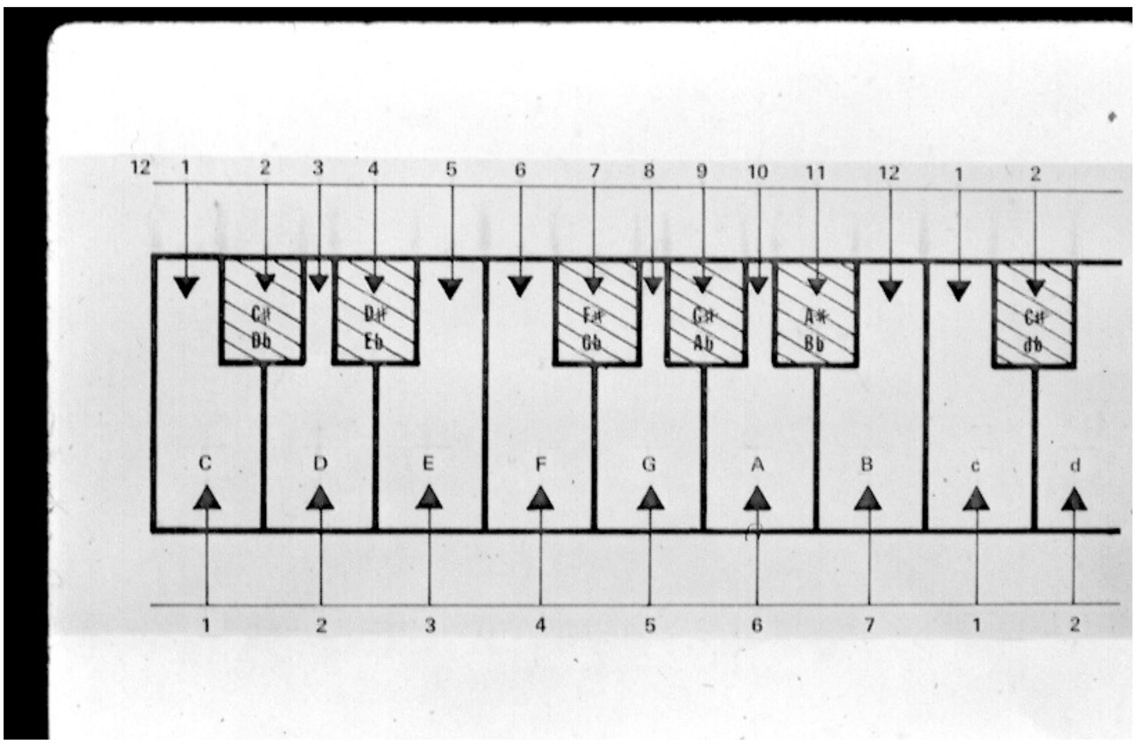
All frequencies henceforth will be expressed in decimal notation and rounded off to tenths.

The sequence above will now be used to construct what is called a Pythagorean scale. The scale to be chosen is called the C Major scale and is the first scale that a beginning piano student would normally encounter because it makes use of the white keys only (Figure 15). (S7)

http://www.culturalmath.com/media/Sound-07.mp3

The first fact that must be taken into account is that scales are normally fitted into the dimension of a single octave. In Figure 15, the octave span to be selected is shown by bold-faced letters. The C Major scale begins with the tone C (the selected note is here called "middle C" on the piano because it roughly separates the set of tones usually played by the left hand from those played by the right hand). On the piano the scale steps to the right in alphabetical order: D, E, F, G. After G, the alphabet commences again, in this case with the standard tone A of 440 cps. This is followed by the tone B and the octave span is completed with the tone shown as lower-case c. All of these tones lie between a C(260.8 cps) and its octave, c(521.6 cps) which is twice its frequency.

Figure 15



To our ears the first tone C and its octave, c, sound melodically identical. This is the reason why men and women can sing the same melody together even though the frequencies of the tones they are singing are usually an octave apart. Without this octave repetition of tones, the transposition of melodies throughout the frequency ranges of orchestral instruments would be an impossibility. The octave with its interval ratio of 1:2 is the simplest harmonic interval and for reasons to be explained later, tones an octave apart in frequency are interpreted as identical for melodic purposes.

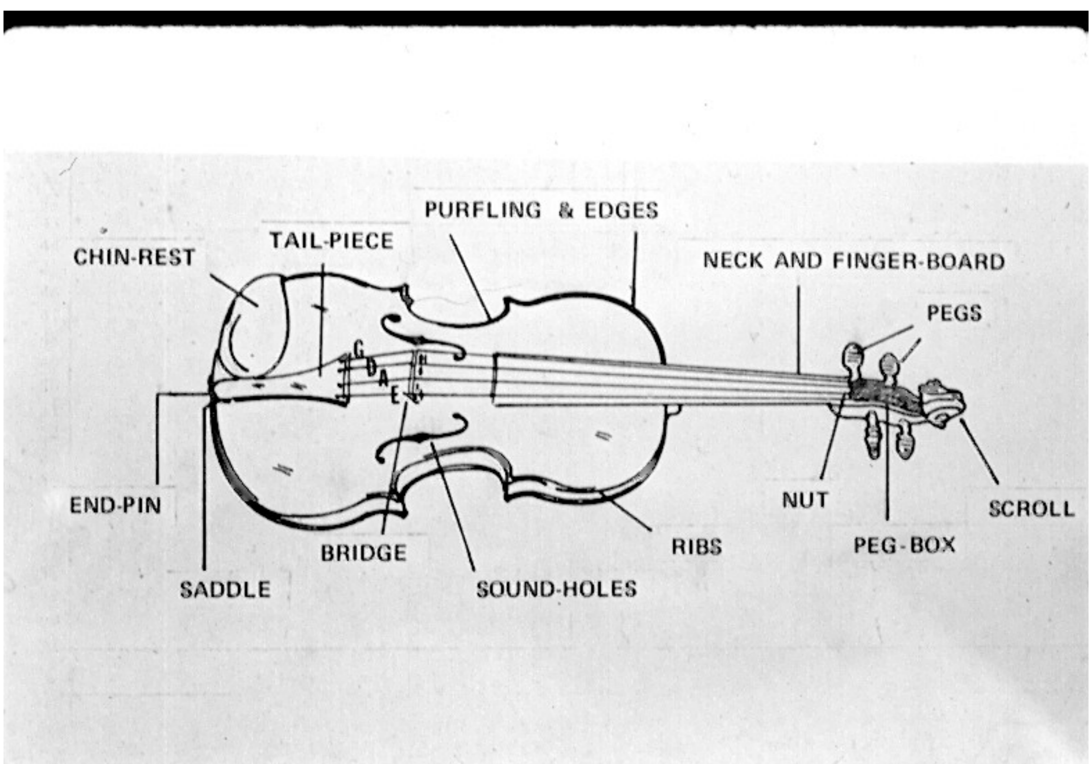
The scale must be constructed with the octave principle in mind. The derivation begins with the standard pitch for A (440 cps). It lies between the two C's. So does D at 293.3 cps (refer to the sequence of fifths above). The other tones in the sequence of fifths must be "transposed" into the octave span. That is, we must find octave replicas of these tones within the frequency boundaries of the two C's. This is achieved by multiplying or dividing the frequencies in the sequence of fifths by appropriate powers of 2, the octave transposing factor. F(86.9) is multiplied by 4 (i.e. 22) to obtain an F at 347.6 cps. C(130.4) is multiplied by 2 to get the bottom tone of the scale 260.8 the frequency of middle C. The octave, c, is gotten by multiplying C(130.4) by 4, resulting in c(521.6). G(195.6) is multiplied by 2 to get 391.2 cps. E(660) is divided by 2 to get 330 cps. Finally, B at (990) is divided by 2 to get 495 cps. The results are shown in an increasing scale of frequencies in the table:

Pythagorean Scale Tone	Letter Designation	Frequency	Derivation
1	C	260.8	130.4 X 2^{1}
2	D	293.3	293.3 X 2^{1}
3	E	330	660/2
4	F	347.6	86.9 X 2^{2}
5	G	391.2	195.6 X 2^{1}
6	A	440	440 X 2^{0}
7	B	495	990/2
8	c	521.6	130.4 X 2^{2}

(Figure 16) illustrates how the tones are notated as notes on the staves of a musical score. The 5th scale tone is the tone G and its numerical designation explains where the name "perfect fifth" comes from. Also note that the scale is essentially completed with the presence of the 8th tone, c, hence the name "octave."

Figure 16

Historically, the C Major scale did not become commonly used in musical composition until the advent of harmony and the system of tonality in the 17th century even though the principles of its construction were understood since the time of Pythagoras (c. 6th century B.C.). Despite the purity of fifths the Pythagorean scale produces, it is not the scale that is commonly used in musical practice today. The reasons for this will be explored later. For the present, we shall return for a look at the violin again (Figure 14).



It can be seen that all of the strings have a vibrating length which extends from the bridge to the nut. The pitch (frequency) of each string can be varied by applying tension at the pegs or by varying the weight of each string according to its basic pitch. Thus, the lower strings are thicker and given added weight by winding with metal. The additional weight allows for tuning the strings to their proper pitches without much differences in the tensions applied. In 1636 the mathematician, Mersenne, incorporated all of these variables into a general law for the vibrations of fixed strings. This law is expressed by the formula f = 1/(2L) SQR(X/M), where f is the frequency of the string; L is the length of the string that vibrates; X is the stretching force or tension on the string; and M is the mass per unit length of the string. As frequently happens in mathematics, certain laws are discovered simultaneously and independently.

This law was named after Mersenne because he published his formula two years before Galileo presented the same results. The part of the formula which deals with the length of the string is sometimes called the Law of Pythagoras, because it was in the course of studying the length of vibrating strings that Pythagoras was reputed to have derived the basic numerical relationships for the musical intervals.

The violinist can check the tuning of his strings by applying the Pythagorean Law. For example (Figure 17), if he presses the A string to the fingerboard at a distance of 1/3 the length of the string from the nut and bows across the remaining 2/3 of the string, the note E will be sounded having the same pitch as the open E string, assuming the E string was tuned correctly by the harmonizing technique described earlier. Thus, 2/3 of the length of the A string gives a tone which has 3/2 of the frequency of A.

Since 2/3 and 3/2 are reciprocals, this shows the frequency of a string is inversely proportional to its length (Figure 18). In actuality, the stopped tone E on the A string will sound different than the open E string, even though they have the same pitch.

Figure 17

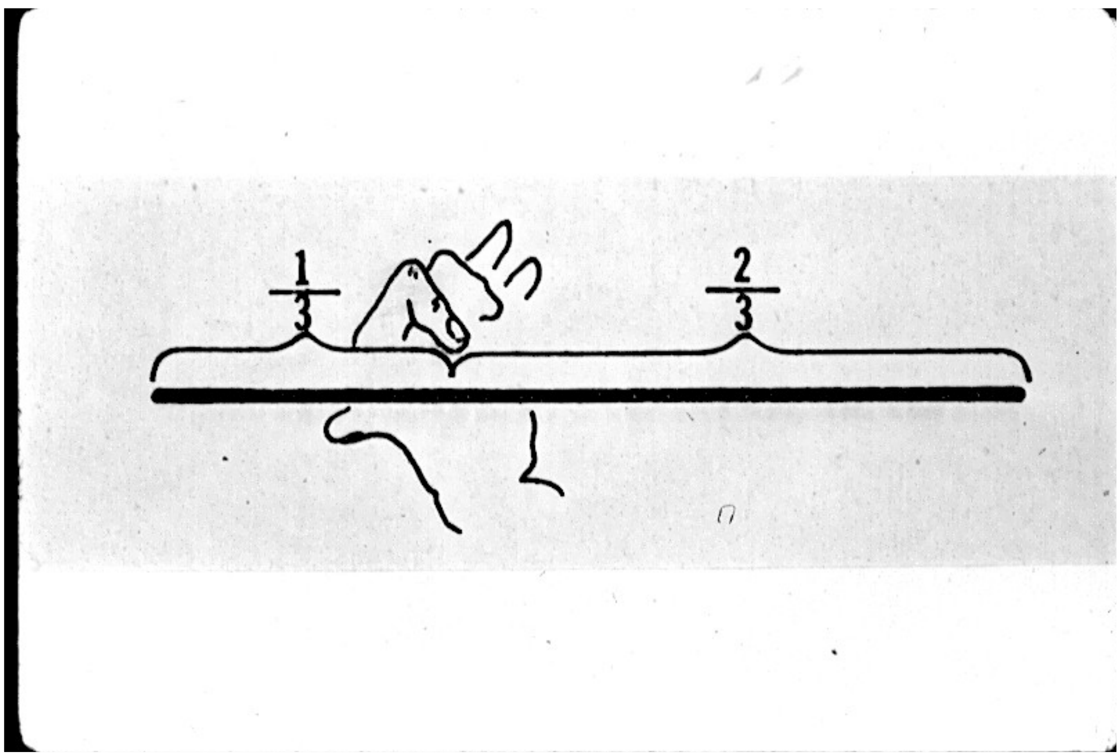
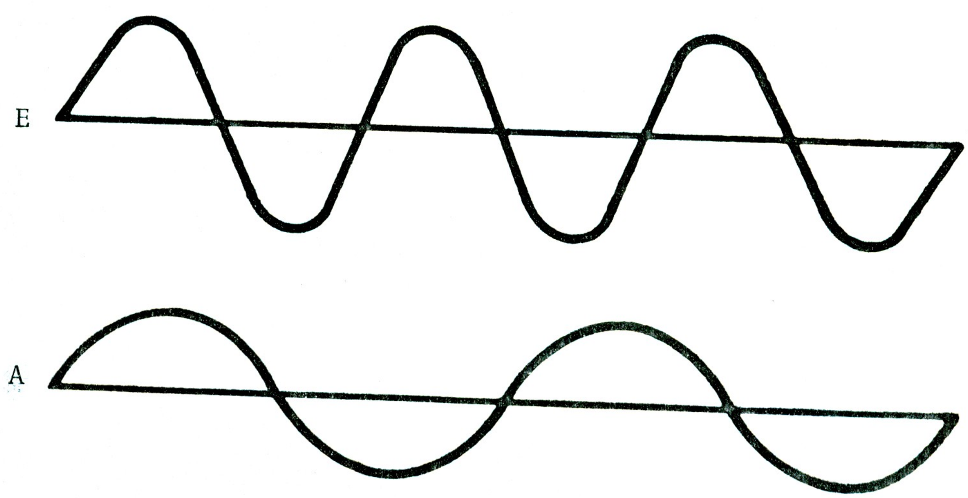


Figure 18

The Fifth Relation shown by sine waves in ratio of 3:2.



The difference we hear is the result of tone quality.(S8)

http://www.culturalmath.com/media/Sound-08.mp3

The open E sounds louder and more vibrant because it is richer in what are called "harmonics" or "overtones." These are tones higher in frequency than the tone E which enter into the production of most musical tones. This occurs because musical instruments are more complicated than a tuning fork in their vibrations. The analysis of a violin string shows that it not only vibrates as a whole, but in parts as well (Figure 19). These vibrating parts are integral fractions of the whole string and by the reciprocal relationship they result in tones being produced that are integral multi-pIes of the fundamental tone being played. Fortunately, the intensity of these bowed violin tones comply with the inverse square law, otherwise they would completely mask the sound of the fundamental tone. The fore-going is summarized in the following table:

THE E STRING AND ITS HARMONICS

Harmonic	Frequency	Ratio	Intensity
1	660	1:1	1/1
2	1320	2:1	1/4
3	1980	3:1	1/9
4	2640	4:1	1/16
5	3300	5:1	1/25
6	3960	6:1	1/36
7	4620	7:1	1/49
8	5280	8:1	1/64

Although the series theoretically goes on to infinity, the law of decreasing intensities soon makes the upper harmonics inaudible.

Figure 19

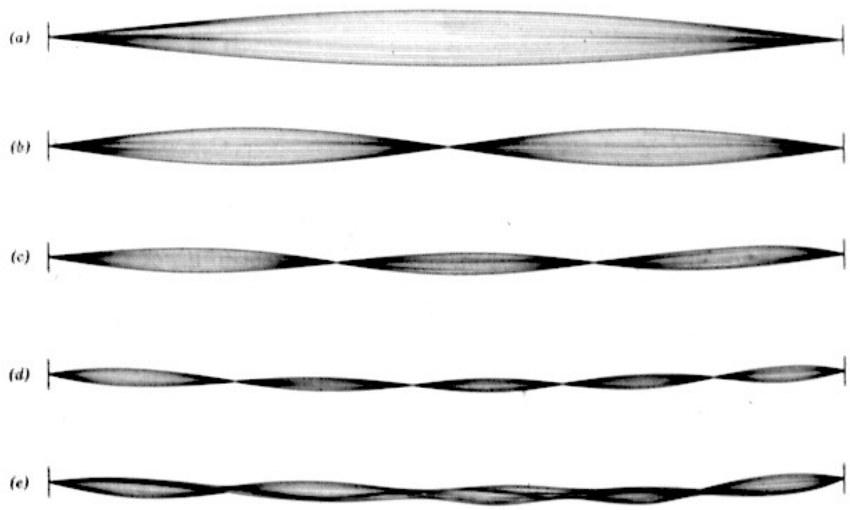


Figure 2.1. Vibration of a string: *(a)* at its fundamental or first harmonic; *(b)* at its second harmonic; *(c)* at its third harmonic; *(d)* at its fifth harmonic; and *(e)* in several harmonics simultaneously.

The resonating box of a violin also contributes its own peculiar vibrations when the strings are set in motion. In fact, it is this characteristic that distinguishes the superb sound of the famous Stradivarius violins. (Figure 20) (S9) (L6): http://en.wikipedia.org/wiki/Stradivarius

http://www.culturalmath.com/media/Sound-09.mp3

Emile Leipps[5] believes that the famous violin makers like Antonio Stradivarius incorporated the Golden Section[6] consciously in the dimension of their instruments. How this may have contributed to their superior sonics is still not understood, but it is an interesting sidelight in the relationship between mathematics and music.

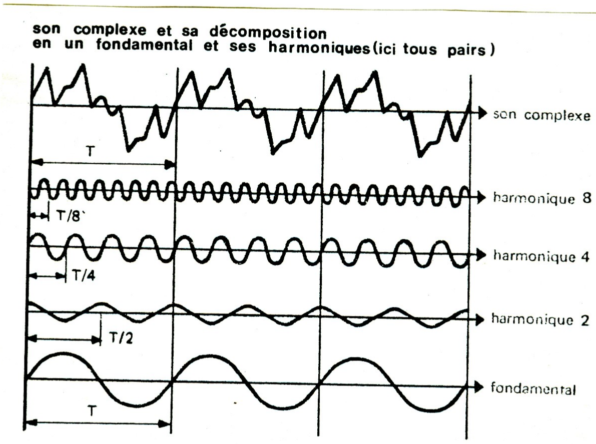
Figure 20

The production of harmonics is also true of a vibrating air column, the essential tone-producing mechanism of the woodwind instruments. All of these additional tones are integrated together with the fundamental pitch to yield what is called "tone color" or "tone quality." Each instrument produces its own characteristic spectrum of harmonics or "partials" as they are also called. This is the reason why the same tone E sounds so different when played by the violin as compared to the oboe. Thanks to the mathematical work of the great French scientist, Joseph Fourier (1768-1839), and the recent developments in electronics, we are now able to visualize the complex wave patterns that are produced by the different instruments. This branch of mathematics is known as "harmonic analysis" and requires a knowledge of trigonometry and calculus. Students in their second year of calculus study Fourier's theorem. (Figure 21) shows the picture of a typical sound wave form as reproduced on an oscilloscope.

L7: http://en.wikipedia.org/wiki/Fourier_analysis

Figure 21

Harmonic analysis of a tone.



We are now in a position to understand why the open E string sounds different than the stopped E fingered on the A string. The open E allows the higher coloring tones to be more audible because the ends of the vibrating string are pressed against the hard material of the bridge and nut. The stopped E has a less well defined node where the softer material of the finger produces more damping of the higher partials. In melodic passages where evenness of tone is desired, care must be exercised by the violinist to finger the notes so that the open strings are avoided. Otherwise these open tones would stand out too much. We are also able to understand more completely why reverberation is such an important aspect of good listening. In a hall with good reverberation, the upper harmonics are more audible and this contributes to proper realization of tone color. This is the essential ingredient for any performance to come alive.

In recent years, experiments in acoustics have verified the importance of another factor in tone quality. Every instrument has a characteristic attack in the tones it produces. This onset. transient, as it is called, is coupled with a decay transient that is specific to each tone an instrument sounds. For example, the brilliant tone of the 'trumpet as compared to the violin is explained by Winkel as follows: "In the case of the trumpet, the first and second partials develop more slowly than the upper partials. This is the reason why the trumpet sounds more clearly defined, with more fundamental, than the violin .... The short onset time of the trumpet (20 milliseconds) permits many more partials ... "[6] It is the presence of the upper partials that are increasingly dissonant with respect to the fundamental that contributes to the "brassy" tones of the trumpet.

As the tuning of the orchestra comes to completion, there is an expectant hush in the hall. The sound of mounting applause signifies the entry of the conductor who bows in acknowledgement and mounts the podium. So much has been written about the role of the conductor that it would be impossible to cover it here. By the time he raises his baton to begin, most of his work has already been accomplished. It is during rehearsals that he communicates his detailed interpretive conception of the work to be played. He must bring into proper balance all of the elements that lead to the realization of the score in sound. At the concert, this integration is conveyed by a series of physically expressed cues which serve to remind the players of what was communicated at the rehearsals. On a higher level, the conductor becomes a physical embodiment of the emotional and philosophical meaning of the music. He is the medium through which the composer "speaks."

Before leaving this chapter, listen to the following two examples of how two composers have integrated the foundations of the discussion above.

In Aaron Copland’s ballet Rodeo, the opening of the slow waltz section imitates the tuning of the orchestra (S10), and the opening of Alban Berg’s violin concerto utilizes the sequence of perfect fifths as a serial motive (S11).

http://www.culturalmath.com/media/Sound-10.mp3

http://www.culturalmath.com/media/Sound-11.mp3

LIST OF WEB SITE LINKS

L1 : http://www.sallepleyel.fr/anglais/la_salle/visite_virtuelle/index.asp

L2 : http://en.wikipedia.org/wiki/Toscanini

L3 : http://en.wikipedia.org/wiki/Paul_VI_Audience_Hall

L4: http://en.wikipedia.org/wiki/Avery_Fisher_Hall

L5 : http://www.exploratorium.edu/music/movies/tuning_hi.html

L6: http://en.wikipedia.org/wiki/Stradivarius

L7 : http://en.wikipedia.org/wiki/Fourier_analysis

LIST OF REFERENCES

Chapter 1 - Acoustics

1. Camp de, The Ancient Engineers, New York: Ballantine Books, 1974, pp. 93-94
2. Ibid., p. 94
3. Wood, Acoustics, New York: Interscience Publishers, 1946, p. 538
4. Jeans, Science and Music, London: Cambridge University Press, 1947, p. 204
5. Leipp, The Violin: History, Aesthetics, Manufacture, and Acoustics, H. Parry, trans., Toronto: University of Toronto Press, 1969
6. Winckel, Music, Sound and Sensation, Thomas Binkley, trans., New York: Dover, 1967, p. 32, 42

LIST OF RECORDED EXCERPTS

S3 : HAYDN: **Sinfonia Concertante CONDUCTOR: ARTURO TOSCANINI**

Mischa Mischakoff, violin; Frank Miller, cello

Paolo Renzi, oboe; Leonard Sharrow, bassoon

March 6, 1948	NBCSO	Studio 8H

RCA 60282 (Vol. 13)
Memories HR 4201

S4: BRAHMS: **Symphony #2 CONDUCTOR: ARTURO TOSCANINI**

Feb. 11, 1952	NBCSO	Carnigie Hall

RCA 60258 (Vol. 7)
RCA 55838 (Red Seal Series Vol. 4, 2 discs)

S9: SIBELIUS: VIOLIN CONCERTO, JASCHA HEIFETZ, VIOLINIST
CHICAGO SYMPHONY, WALTER HENDL, COND.
RCA VICTOR COMPACT DISC RCD1-7019

S10: COPLAND: RODEO BALLET SUITE, SLOW WALTZ
NEW YORK PHILHARMONIC, LEONARD BERNSTEIN, COND.
COLUMBIA LP SET MG 30071

S11: BERG: VIOLIN CONCERTO, ITZHAK PERLMAN, VIOLINIST
BOSTON SYMPHONY, SEIJI OZAWA, COND.
DEUTSCHE GRAMMOPHON CASSETTE 3301 110